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ABSTRACT

The current period in mathematics education can be characterized as one of reform. Many feel that children in the United States are not learning enough appropriate mathematics; these critics are concerned with the specific areas of problem solving and children's conceptions of the nature and uses of mathematics. A pretest/posttest experimental design study examined the effects of SQUARE ONE TV, a television series about mathematics aimed at 8- to 12-year-old children, on the problem-solving behavior and attitudes toward mathematics of 240 fifth graders from 4 public schools in Corpus Christi, Texas. Performance and attitude data were collected from a subgroup of 24 students exposed to 30 SQUARE ONE TV programs and from 24 students in a control group having no SQUARE ONE TV contact. Reported here are the purpose and general design of the study and the effects of SQUARE ONE TV on children's problem solving," presented in the first two volumes of a five volume report. Results on children's problem-solving actions indicated that viewers of SQUARE ONE TV programs demonstrated statistically significant gains between the pretest and posttest, both in their use of problem-solving behaviors and in the mathematical completeness and sophistication of their solutions, and that this effect was not different for children of differing sex, socioeconomic status, ethnicity, or standardized test performance. The overall implication is that sustained, unaided viewing of SQUARE ONE TV can have a significant impact on children's problem solving. (MDH)

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CHILDREN'S PROBLEM-SOLVING BEHAVIOR AND THEIR ATTITUDES TOWARD MATHEMATICS:
A STUDY OF THE EFFECTS OF SQUARE ONE TV

Volume I Introduction: Purpose and General Design of the Study

Volume II The Effects of Square One TV on Children's Problem Solving

Children's Television Workshop

New York, 1990

The production of SQUARE ONE TV and the research reported here have been supported by the National Science Foundation, the Corporation for Public Broadcasting, the Carnegie Corporation, and the U.S. Education Department. First season production was also supported by the Andrew Mellon Foundation and by the International Business Machines Corporation.

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Children's Problem-Solving Behavior and Their Attitudes toward Mathematics: A Study of the Effects of **SQUARE ONE TV**

VOLUME I

Introduction: Purpose and General Design of the Study

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New York, 1990

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PREFACE

This is the first of five volumes that describe an evaluation entitled "Children's Problem-Solving Behavior and Their Attitudes toward Mathematics: A Study of the Effects of **SQUARE ONE TV**." The study was designed to assess the effects of **SQUARE ONE TV** on children's use of problem-solving actions and heuristics and on their attitudes toward mathematics. In addition, children were interviewed about their opinions of and reactions to **SQUARE ONE TV** itself.

The contents of the five volumes are as follows:

Volume I:	Introduction: Purpose and General Design of the Study
Volume II:	The Effects of **SQUARE ONE TV** on Children's Problem Solving
Volume III:	Children's Attitudes toward Mathematics and the Effects of **SQUARE ONE TV**
Volume IV:	The **SQUARE ONE TV** Interview: Children's Reactions to the Series
Volume V:	Executive Summary

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CHAPTER 1

INTRODUCTION

SQUARE ONE TV is a television series about mathematics, aimed at 8- to 12-year-old children. Produced by the Children's Television Workshop (CTW), the series was developed in accordance with the "CTW model." This interdisciplinary approach to television production brings together content experts, television production specialists, and educational researchers, who collaborate throughout the life of the project.

This chapter provides a brief description of **SQUARE ONE TV**, relevant issues in mathematics education, and the purposes of the present study.

A Context of Mathematics Education

The creation of **SQUARE ONE TV** must be considered within a context of present-day mathematics education. Forces within the mathematics education community have influenced the direction that the series has taken, as well as its evaluation.

The current period in mathematics education can be characterized as one of reform. There is widespread dissatisfaction among professionals and laypeople alike with the present state of mathematics learning among children in the United States (Hill, 1987; McKnight, Crosswhite, Dossey, Kifer, Swafford, Travers, & Cooney, 1987; National Assessment of Educational Progress, 1988; National Council of Teachers of Mathematics, 1989; National Research Council, 1989; Romberg, 1984); indeed, much of the literature refers to the deficiencies that currently exist as a "crisis" in mathematics education. Many feel that children in the United States are not learning enough of the mathematics that will be appropriate for the coming decades, and that children's conceptions of the nature and uses of mathematics are inaccurate and misinformed. Among the areas of specific concern are these two:

Concern with problem solving. American elementary school students typically demonstrate low levels of achievement on tasks other than simple arithmetic or one-step word problems. This is not surprising, considering that many of the most widely used elementary school mathematics curricula place an overwhelming emphasis on computational skills and very low-level kinds of routine word problems. Despite the recognition among mathematics educators that problem solving must occupy a central role in mathematics curricula, the emphasis on basic calculation persists in the classroom, in textbooks, in standardized tests, and also in parental expectations.

Concern with attitudes. Beginning in the high school years when state requirements for mathematics courses are no longer applicable, the attrition rate in mathematics courses increases sharply. But students' attitudes toward mathematics, which are major contributors to this wholesale flight from secondary school mathematics (NRC, 1989, pp. 9-11), are rooted in their earlier experiences with and impressions of the subject. Mathematics educators agree that efforts must be made to enhance children's enjoyment of mathematics, their motivation in using mathematics, and their beliefs about its nature and utility, beginning as early as the elementary school grades.

The roles of NCTM and MSEB. Two of the principal professional groups in the field, the National Council of Teachers of Mathematics (NCTM) and the Mathematical Sciences Education Board (MSEB), have been increasingly active in articulating the problems besetting mathematics education and in offering fairly detailed suggestions for ameliorating the situation. The MSEB reports entitled Everybody Counts (NRC, 1989) and Reshaping School Mathematics: A Philosophy and Framework for Change (NRC, 1990) provide a detailed compendium of the problems affecting U.S. mathematics education and an overall conceptualization of what needs to be done. NCTM's Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) describes more specifically a vision of pre-college mathematics and its assessment. In keeping with the MSEB position, the NCTM Standards document is based on the premise that "all students need to learn more, and often different mathematics... [and that] instruction in mathematics must be significantly revised" (NCTM, 1989, p.1). To this end, NCTM has identified new goals for students: that they learn to value mathematics, become confident in their ability, become mathematical problem solvers, and learn to communicate and reason mathematically.

The Goals of **SQUARE ONE TV**

SQUARE ONE TV was designed in an attempt to contribute to the reform effort. The series has three main goals:

I. To promote positive attitudes toward, and enthusiasm for, mathematics;

II. To encourage the use and application of problem-solving processes; and

III. To present sound mathematical content in an interesting, accessible and meaningful manner.

Each of these goals is refined into a range of subgoals; the complete breakdown is fully explicated in the goals statement presented in Appendix I.A.

The goals of **SQUARE ONE TV** go beyond those of the typical elementary school curriculum. In contrast to standard mathematics instruction, **SQUARE ONE TV** is based on the modeling of problem-solving actions and heuristics (Goal II) and the presentation of a broad range of mathematical content that includes topics not usually explored in the elementary grades (Goal III). Even standard topics such as arithmetic are typically presented in the context of problem solving. At the same time, the series is designed to support and stimulate interest in mathematics -- to demonstrate its utility, its beauty, and its accessibility to non-experts (Goal I). In fulfilling these goals, **SQUARE ONE TV** attempts to supplement school mathematics by exposing children to a wide array of mathematical ideas.

In emphasizing the importance of encouraging problem solving and positive attitudes toward mathematics, the goals of **SQUARE ONE TV** are consonant with those of NCTM. The underlying philosophy of the series is thus in keeping with the ongoing reform movement in mathematics education.

The Audience

SQUARE ONE TV is designed for 8- to 12-year-old children to view at home. Secondary audiences for the series include teachers and parents of the target audience, as well as other children and adults.

The series is generally broadcast in the afternoons, Monday through Friday, on public television stations; it is also included by some stations as part of their instructional television (ITV) schedule. Although most viewing of the program occurs at home, some teachers have used it in school as well.

The Format

Three seasons of **SQUARE ONE TV** have been produced as of January, 1990, resulting in a total of 155 programs. Each program is one-half hour in length, and employs what is called a "magazine" format, in which a variety of discrete segments constitute an entire program. The series is humorous, and the majority of segments parody television styles and conventions familiar to children. Several types of segments are used: studio sketches (that feature a seven-player repertory company), game shows, short films, music videos, and animation. Each program of **SQUARE ONE TV** ends with an installment of "Mathnet" (a serialized parody of the detective series "Dragnet"), in which two mathematicians use mathematical problem solving to solve crimes.

The Role of CTW Research

CTW research on **SQUARE ONE TV** can be divided into two main types: formative and summative. Researchers have used these terms in a variety of ways. For our purposes, "formative research" is a continuing line of research conducted before and during the production of material for the series. The aim of formative research is to assess the preferences and needs of **SQUARE ONE TV**'s target audience and to use them to help shape the production of the series; such research provides the production and content staffs with feedback on the comprehensibility and appeal of proposed material.

By contrast, summative research (such as the present study) assesses the impact of **SQUARE ONE TV** after production has been completed; its aim is to examine whether the series has been successful in meeting its goals (although, naturally, this sort of research also holds formative implications for future production). Two previous summative studies are described here.[1]

Previous CTW Research Concerning SQUARE ONE TV

One summative study, "The Comprehension and Problem-Solving Study" (Peel, Rockwell, Esty, & Gonzer, 1987), examined children's comprehension of a sample of ten segments taken from the first season of the series; the segments contained a wide variety of mathematical content, but all concerned problem solving. Comprehension was assessed on three levels: recall of the problems and solutions shown, understanding of the mathematical principles underlying but not fully explicated in the segments, and extension of concepts shown in the segments to related problems.

The study found the segments' mathematical content to be accessible to children across

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[1] Of course, there is also a body of more general research in the mathematics education literature which holds implications for the creation and evaluation of **SQUARE ONE TV**. We will discuss research pertaining to mathematical problem solving in Volume II and research pertaining to attitudes about mathematics in Volume III.

the series' target age range; third through sixth graders were able to isolate and recall both the problems and solutions which had been shown. Also, the content appeared to be age-appropriate. Well over half of the third graders gave satisfactory answers to questions which assessed their understanding of the segments' underlying mathematical content, and performance increased to over 80% for sixth graders; thus, it seemed that the segments were neither so difficult that no one could understand them nor so easy that third graders performed as well as sixth graders. Moreover, many children were able to extend the mathematical principles used in the segments to solve related problems that had not been shown. Finally, the study demonstrated that viewers perceived the segments' characters as "happy" or "proud" because they had solved problems or because they had demonstrated competence in doing so.

A second study, referred to as "The Mathnet Study" (Schauble & Peel, 1987), focused on **SQUARE ONE TV**'s recurring "Mathnet" format. The main purpose of the study was to determine whether "Mathnet" could provide a context for children's informal problem solving. In addition, it assessed children's conceptions of what mathematics is.

The results of the study suggested that "Mathnet" could indeed serve as a springboard for children to engage in informal problem solving; third through sixth graders attempted to solve "Mathnet" mysteries themselves by generating hypotheses and testing them with evidence they extracted from the series. With regard to children's conceptions of mathematics, the study found that children were able to identify numerous activities appearing in "Mathnet" as examples of mathematics.

Taken together, then, these two studies demonstrate that: children across **SQUARE ONE TV**'s target age range can adequately grasp the various mathematical content presented on the series, its content is age-appropriate, and it can provide a context for children's informal problem solving. In addition, children see characters who solve problems in the series as being pleased with their own competence and accomplishments.

Purposes of the Present Study

The present study builds upon this previous research in several ways. While each of the two studies described above examined problem solving in the context of problems presented in **SQUARE ONE TV**, the current study presents children with new problems to see how the series might affect problem solving in a more general sense. While children in "The Comprehension and Problem-Solving Study" were asked to assess the feelings of the characters shown in the segments, the present study examines the changes in the children's own attitudes that might result from viewing **SQUARE ONE TV**. And while "The Mathnet Study" assessed children's conceptions of what mathematics is, the present study examines how those conceptions might be affected by exposure to **SQUARE ONE TV**. In this way, the current study provides a direct, experimental test of the series' attainment of Goals I and II; that is, it attempts to describe the changes in children's attitudes toward mathematics and their use of problem-solving techniques that might arise as a result of sustained viewing of **SQUARE ONE TV**.

With regard to Goal I of the series, the study is designed to explore children's attitudes toward mathematics. Here, we have conceived of "attitude" as pertaining to issues of motivation, enjoyment, perceptions of usefulness and importance, and children's conceptions of what mathematics is, i.e., their "construct" of mathematics. The study attempts, first, to provide a description of each of these aspects of children's attitudes toward mathematics, and second, to examine the degree to which **SQUARE ONE TV** can influence those attitudes. With regard to Goal II, the study examines the impact of **SQUARE ONE TV** on children's problem-solving actions (particularly problem treatment and problem follow-up) and the extent to which they use a variety of heuristics (e.g., constructing tables or graphs, looking for patterns, or working backwards) in problem solving. Further, the study assesses the impact of the series on the mathematical completeness and sophistication of children's solutions to nonroutine problems.

Clearly, the focus of this evaluation is closely tied to the goals of **SQUARE ONE TV**.

Because of this close correspondence between series goals and evaluation, the present study serves as an example of what NCTM has termed "alignment" (NCTM, 1989, pp. 193-195). The NCTM Standards report stresses the importance of insuring that an evaluation of a curriculum is closely aligned with the nature of the curriculum being evaluated. That is, the goals of the curriculum must guide both the design of the evaluation and the construction of the instruments used. This idea of alignment between program and evaluation was consciously incorporated into the present study, as will become more apparent later in this and subsequent volumes.

CHAPTER 2

METHOD

The purpose of this chapter is to describe the sample, design, treatment, procedure, and instruments used in this study.

Sample

The subjects who participated in this study were fifth graders taken from four public schools in Corpus Christi, Texas. The site was chosen because it is one of the few cities in the country in which **SQUARE ONE TV** had not been part of the regular public television broadcast schedule prior to completion of data collection. Although it had been part of the ITV schedule, it had not been shown by teachers in the participating schools. (After the study was completed, the series was added to the regular broadcast schedule.)

The schools included in the study were chosen because all four employ a standard, district-wide mathematics curriculum and use a traditional mathematics textbook, the Addison-Wesley series (Eicholz, O'Daffer, and Fleenor, 1985). Based on our knowledge of a variety of mathematics curricula, we consider this curriculum and the Addison-Wesley text to be fairly typical of those found throughout the country.

Schools were matched as pairs on the basis of students' socio-economic status (SES), standardized achievement test scores, racial/ethnic composition, general curriculum and the textbooks used (mathematics and other subjects). One school in each pair was randomly designated as a "viewing" school, and the other was designated as a "nonviewing" school. Children in viewing schools were shown **SQUARE ONE TV** and children in nonviewing schools were not.

The total sample consisted of 240 children, 48 of whom participated in the main part of the experiment (i.e., the Problem-Solving Activities and Attitude Interview, as described in the Procedure and Instrumentation section below). The 240 children comprised all of the students in the 11 regular fifth grade classrooms in the four schools. All eleven classrooms were represented in the subsample of 48 children.

The subsample consisted of 12 children per school. The selection of these children was accomplished in two stages. In the initial stage, three criteria for selection were employed. First, because pre- and posttests would be conducted over the course of two months, children were not included in the main part of the experiment if they had shown a high degree of mobility (i.e., if they had moved four or more times) in the two previous school years. Second, since the procedure used in the main part of the experiment consisted largely of interviews, children were also excluded if they were currently enrolled in an English as a Second Language program. Third, the availability of achievement test scores allowed us to select a sample that included high, medium, and low achievers as well as children from low to middle SES backgrounds.

In the second stage of selection, children within the subsample were individually matched as pairs on gender, race/ethnicity, achievement test scores, and eligibility for free lunch (an indicator of SES).

In all, 24 boys and 24 girls participated in the main part of the experiment. 29% were Anglo, 4% were African-American, and 67% were Latino; these percentages mirrored those found in the local school system as a whole.

Additionally, after the viewers had been exposed to **SQUARE ONE TV** for five weeks, all 240 children (114 viewers and 126 nonviewers) were asked to write essays designed to assess their attitudes toward jobs which involve mathematics. Of these 240 children, 52% were boys and 48% were girls; 28% were Anglo, 5% were African-American, and 67% were Latino.

Design

As Table 1 illustrates, the basic design of the study involved two groups of children, designated as "viewers" or "nonviewers." The subsample of 48 children was tested before and after the viewers were exposed to **SQUARE ONE TV**.

Table 1

Basic design of the study

VIEWERS	NONVIEWERS
Pretest	Pretest
View 30 programs of **SQUARE ONE TV**	Do not view
Posttest	Posttest

The main part of the pretest and posttest took the form of task-based interviews (Davis, 1984). These interviews assessed children's use of problem-solving actions and heuristics and their attitudes toward mathematics. By comparing the changes in the two groups' performance over time, we could determine whether pretest-posttest changes in viewers' performance were a function of their having watched **SQUARE ONE TV**.

Blindness. A critical concern in designing this study was to insure that the experimenters who interviewed children and/or coded their behavior would not inadvertently bias the results of the study in any way (e.g., by probing viewers' responses more deeply than nonviewers'). For this reason, all interviewers and coders were unaware of whether individual children had been designated as viewers or nonviewers; that is, both interviewers and coders were blind as to the children's experimental condition. Thus, the data obtained in the study do not reflect expectations of the interviewers or coders.

Treatment

All of the students in the two viewing schools were exposed to six weeks (30 programs) of **SQUARE ONE TV**, one program per day, during school hours. More detail on the viewing schedule can be found in the Procedure section below.

One half of the 30 programs presented to the children were taken from the first production season of **SQUARE ONE TV**, and the other half came from the second season. The programs were selected so that no "Mathnet" segments were repeated in the course of the six weeks, and repetition of other **SQUARE ONE TV** segments was kept to a minimum. A list and content analysis of the programs included in the treatment can be found in Appendix I.B; summary descriptions of the mathematical content presented in the 30 programs can be found in Appendix I.C.

A critical feature of the design of this study was that the teachers in the viewing schools did not alter their usual mathematics instruction in any way. They did not use **SQUARE ONE TV** as part of their teaching, they did not comment on it, and they did not draw any connections for the children between the series and mathematics. Thus, although viewers were shown **SQUARE ONE TV** in school, their exposure to the series consisted of sustained, unaided viewing.

In addition, teachers were instructed not to show **SQUARE ONE TV** during mathematics class; rather, it was shown during free periods, recess, social studies, and/or science class, as determined by the school principals on the basis of the children's class schedules. This accomplished three purposes: It allowed viewers to receive their usual type and amount of classroom mathematics instruction (i.e., the same amount of mathematics instruction as nonviewers received), it helped to prevent teachers from incorporating **SQUARE ONE TV** into their mathematics instruction, and it lessened the chances that children might think of **SQUARE ONE TV** as a part of their mathematics curriculum.

Children in the two nonviewing schools did not see **SQUARE ONE TV** at all, and their instructional schedule was left unchanged.

Procedure and Instrumentation

Preliminary Meetings. In the weeks before the study began, one experimenter (who did not administer either the problem-solving or attitudinal measures) met with the teachers and principals of the schools included in the study. Teachers were asked not to draw any connection for children between the interviews and either mathematics or **SQUARE ONE TV**. For example, they were asked not to refer to interviewers as "the people from **SQUARE ONE TV**" or mention that interviewers would be talking to the children about mathematics. Indeed, posttest interviews made it clear that children had not made the connection between the interviews and **SQUARE ONE TV**, as will be discussed at greater length in Volume IV.

Similarly, as discussed in the Treatment section above, teachers and principals were asked not to incorporate the material shown in **SQUARE ONE TV** into their mathematics instruction, to insure that any effects observed in the posttest would truly be effects of viewing **SQUARE ONE TV** and not of any modified school curriculum.

In addition, it was explained to teachers and principals that to prevent interviewers from inadvertently biasing their results, the interviewers would not know which children were watching **SQUARE ONE TV** (i.e., that they would be blind to the children's experimental condition) and that it was important for interviewers to remain blind.

To help insure that the interviewers would remain blind, assistants were recruited through the local Board of Education. If scheduling problems arose (e.g., if a child failed to appear on time for an interview session), these assistants served as go-betweens to prevent any possible communication between interviewers and teachers.

Also during this period, permission slips were sent home to be signed by the parents of prospective subjects. Again, to prevent children from drawing a connection between the study and **SQUARE ONE TV**, the slips made no mention of either the series or mathematics; the study was described solely as an investigation of children's problem solving.

Overview of the Study. Four instruments were used in the study: (a) the Problem-Solving Activities (PSAs), (b) the Attitude Interview, (c) the Essay, and (d) the **SQUARE ONE TV** Interview. The first two measures made up the main part of the experiment and were used with the subsample of 48 children; the third measure was used with all 240 children in the sample; and the fourth was used only with children who were shown **SQUARE ONE TV**. The four instruments were administered according to the schedule shown in Table 2. We will now describe each aspect of the study in greater detail.

The Preptalk. The Week 1 procedure began one day before the pretest when the 12 subsample children at a given school were gathered together for an introductory "Preptalk." The Preptalk introduced children to the study and described the basic nature of the interview process. No mention was made of either mathematics or **SQUARE ONE TV**; children were told only that the interviewers were interested in finding out how children "think about things." It was explained that the children would be participating in a non-directive interview, in which interviewers "often ask for more detail or explanation because it is hard to get inside of your head to know exactly what you are thinking.... When we ask questions, it does not mean that we approve or disapprove of what you are saying." To further illustrate this point, two interviewers acted out a brief mock interview, demonstrating that no directive feedback was given at any point, regardless of whether the subject's statements were correct or incorrect.

Table 2

Schedule of the study

Date (1989)	Experimental Group (Viewers)	Control Group (Nonviewers)
Week 1 (2/20-24)	Preptalk and Pretest (PSAs & Attitude Interview) (24 children)	Preptalk and Pretest (PSAs & Attitude Interview) (24 children)
Weeks 2-9* (3/6-4/28)	View 30 programs of **SQUARE ONE TV** (114 children)	No viewing (126 children)
Weeks 8 & 9 (4/17 & 4/24)	Essay (114 children)	Essay (126 children)
Week 10 (5/1-5/4)	Posttest (PSAs & Attitude Interview) (24 children)	Posttest (PSAs & Attitude Interview) (24 children)
(5/5 & 5/8)	**SQUARE ONE TV** Interview (24 children)	---

* Note: There were two weeks during this eight-week period when **SQUARE ONE TV** was not shown: the week of March 20 (which was the children's Spring break) and the week of April 3 (when standardized achievement tests were administered throughout the school district).

Thus, the Preptalk stressed that the children should not think of the elaborate probing used during the interviews as any sort of feedback. It was hoped that this would help prevent children from viewing interviewers' probe questions as an indication that they had given a wrong answer.

Finally, children were allowed to examine a kit of materials containing various items (e.g., markers, a calculator, a ruler) that would be available to them while they worked on the three Problem-Solving Activities. They were told that some of the items in the kit might be useful and some might not, but that all would be available if they wanted them.

Interested readers can find the complete protocol used for the Preptalk in Appendix I.D.

Pretest. The pretest was conducted individually with 48 children; it required two 55-minute sessions per child. In Session 1, children worked on two Problem-Solving Activities. Session 2 (conducted with a different interviewer on the next day) consisted of a third Problem-Solving Activity and the Attitude Interview. All interview sessions were conducted in a private room provided by the school and were videotaped and audiotaped for subsequent transcription and analysis.

The Problem-Solving Activities (PSAs). The PSAs are a range of mathematically rich problem situations that allow the child to demonstrate the problem-solving actions described by Goal II and that permit a number of different approaches to reaching solutions. Their purpose is to measure the number and variety of problem-solving actions and heuristics that children use, as well as the mathematical completeness and sophistication of the solutions they reach.

In each PSA, the researcher introduced a problem and left the child to work on it by himself or herself. No guidance was given as to how the child might solve the problem. Then, after the child was given the opportunity to work on the PSA, the researcher used a series of probe questions to find out what the child was thinking during the work session. Special emphasis was placed upon having the child describe and assess the choices he or she made during the problem-solving process.

Three PSAs were used in this study. The least complex problem, PSA A, is a combinatorics problem involving circus performers or stripes on a shirt. PSA B (a problem of medium complexity) involves sorting party guests or price tags into piles that meet several conditions. And PSA C (the most complex PSA) asks children to determine what is wrong with a complicated mathematical game and to fix it. Further information about the content of the PSAs can be found in Chapter 3 of Volume II, which describes each PSA in detail.

To enable us to assess the impact of **SQUARE ONE TV**, children were given similar problems in the pretest and posttest. However, to minimize the effects of practice and repetition, the pretest and posttest problems were not identical. Thus, two versions of each PSA were devised; these are referred to as PSAs A and A', B and B', and C and C'. PSA C', for example, is similar to PSA C in that it asks the child to determine what is wrong with a mathematical game, but both the materials used in the game and the changes needed to fix it are somewhat different from those used in PSA C (although the underlying mathematical principles are the same). Extensive pilot testing showed difficulty to be roughly equivalent within each pair of PSAs (e.g., PSAs C and C' are about equally difficult). Half of the children were presented with PSAs A, B, and C in the pretest; the other half were given the equivalent PSAs A', B', and C' in the pretest. The complementary versions were then administered in the posttest.

Children's performances were coded along two dimensions: (a) The number and variety of problem-solving actions and heuristics they used (as described in Goal II of **SQUARE ONE TV**) and (b) the mathematical completeness and sophistication of their solutions. This coding took into account two sources of evidence: One source was the behaviors that the children visibly performed while working on a given PSA. The other was the (often internal) processes which children later reported having used when they were interviewed immediately after working on the PSA. All aspects of the PSAs will be discussed more fully in Volume II.

The Attitude Interview. The purpose of the Attitude Interview was to address Goal I of the series ("To promote positive attitudes and enthusiasm for mathematics"). It was designed both to explore children's attitudes toward mathematics and to evaluate the impact of **SQUARE ONE TV** upon those attitudes. Here, we have conceived of "Attitude" as pertaining to issues of motivation, enjoyment, perceptions of usefulness and importance, and children's conceptions of what mathematics is, i.e., their "construct" of mathematics.

The aims described under the Goal I subgoals guided our creation of the specific interview questions used. Goal IA ("Mathematics is a powerful... tool, useful to solve problems... and to increase efficiency") provided the impetus for interview questions that assessed children's construct of mathematics and the degree to which children think of mathematics as useful and important. Goal IB ("Mathematics is beautiful and aesthetically pleasing") resulted in questions about the children's enjoyment of mathematics. And Goal IC ("Mathematics can be used, understood, and even invented by non-specialists") led to questions assessing children's motivation in using mathematics. The questions were open-ended and asked with respect to three domains: (a) the problem-solving activities which children engaged in as part of the study, (b) problem solving in general, and (c) mathematics in and out of school.

The Attitude Interview consists of open-ended questions aimed at developing an elaborate picture of individual children's beliefs and feelings regarding mathematics; thus, interviewers were trained to ask follow-up probe questions to draw out and reveal the full complexity of the issues children raised. Detailed coding schemes were developed through an analysis of the children's responses. All aspects of the Attitude Interview are described in greater detail in Volume III.

Exposure to **SQUARE ONE TV.** Over the course of eight weeks following the pretest, viewers were shown 30 programs of **SQUARE ONE TV**. Specific details concerning this exposure have been described in the Treatment section above.

Children in the two nonviewing schools did not view **SQUARE ONE TV** at all during this period; their schedule did not change from what it usually was.

The Essay. The purpose of the Essay was to provide some insight into the attitudes of the entire sample. This assessment was necessarily less detailed than that provided by the Attitude Interview, in which interviewers could, for example, ask children to clarify any unclear responses. A variety of practical constraints allowed us to use the Attitude Interview with only 48 of the 240 children in our sample, whereas the Essay could be used with all 240. The Essay was administered to all viewers and nonviewers after the viewers had been shown five weeks of **SQUARE ONE TV**.

In the Essay, children were asked to write essays explaining why they would or would not want a job involving mathematics. The essays were coded along several dimensions to lend insight into the children's conceptions of mathematics, as reflected in the types and uses of mathematics discussed and the range of occupations considered to be "mathematics jobs."

The Essay was administered by the children's regular teachers in class; the teachers were instructed to treat the measure as though it were a routine in-class writing exercise rather than a part of our study. In addition, to prevent children from being surprised by the Essay and possibly linking it to the study, a second, "buffer" essay was designed. The buffer essay, which concerned the relative importance of learning in school vs. learning from sources outside school (e.g., family, friends, books), was administered one week before the Essay. Because the purpose of the buffer essay was simply to accustom children to the task used in the Essay, and because we had no experimental predictions about children's responses to the buffer essay, the data from the buffer essay were not analyzed.

The Essay will be discussed in greater detail in Volume III.

Posttest. The procedure used in the posttest was essentially identical to that used in the pretest. The posttest was conducted with the same 48 children who had participated in the pretest; it required two 55-minute sessions, the first consisting of two PSAs and the second consisting of one PSA and the Attitude Interview. Children met with the same interviewers who had interviewed them in the pretest.

Those children who had completed PSAs A, B, and C in the pretest were presented with PSAs A', B', and C' in the posttest; the opposite was true for those who had been given the latter set of PSAs in the pretest. The posttest Attitude Interview was essentially identical to that used in the pretest.

The **SQUARE ONE TV** Interview. The purpose of the **SQUARE ONE TV** Interview was to explore children's perceptions of **SQUARE ONE TV** in an attempt to provide further insight into the ways in which the series might have affected their conceptions of mathematics and problem solving. The interview included questions concerning the series' educational value and appeal, the children's reactions to particular formats, and their appreciation of the mathematics presented in those formats. This was the final instrument used in the study.

The 30-minute interview was individually administered to all of the 24 viewers in the subsample one or two school days after the completion of the posttest. The interview will be described in greater detail in Volume IV.

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APPENDIX I.A:

Goals of **SQUARE ONE TV**

ELABORATION OF GOALS OF SQUARE ONE TV

GOAL I. To promote positive attitudes toward, and enthusiasm for, mathematics by showing:

A. Mathematics is a powerful and widely applicable tool useful to solve problems, to illustrate concepts, and to increase efficiency.

B. Mathematics is beautiful and aesthetically pleasing.

C. Mathematics can be understood, used, and even invented, by non-specialists.

GOAL II. To encourage the use and application of problem-solving processes by modeling:

A. Problem Formulation

1. Recognize and state a problem.
2. Assess the value of solving a problem.
3. Assess the possibility of solving a problem.

B. Problem Treatment

1. Recall information.
2. Estimate or approximate.
3. Measure, gather data or check resources.
4. Calculate or manipulate (mentally or physically).
5. Consider probabilities.
6. Use trial-and-error or guess-and-check.

C. Problem-Solving Heuristics

1. Represent problem: scale model, drawing, map; picture; diagram, gadget; table, chart; graph; use object, act out.
2. Transform problem: reword, clarify; simplify; find subgoals, subproblems, work backwards.
3. Look for: patterns; missing information; distinctions in kind of information (pertinent or extraneous).

4. Reapproach problem: change point of view, reevaluate assumptions; generate new hypotheses.

D. Problem Follow-up

1. Discuss reasonableness of results and precision of results.
2. Look for alternative solutions.
3. Look for alternative ways to solve.
4. Look for, or extend to, related problems.

GOAL III. To present sound mathematical content in an interesting, accessible, and meaningful manner by exploring:

A. Numbers and Counting

1. Whole numbers.
2. Numeration: role and meaning of digits in whole numbers (place value); Roman numerals; palindromes; other bases.
3. Rational numbers: interpretations of fractions as numbers, ratios, parts of a whole or of a set.
4. Decimal notation: role and meaning of digits in decimal numeration.
5. Percents: uses; link to decimals and fractions.
6. Negative numbers: uses; relation to subtraction.

B. Arithmetic of Rational Numbers

1. Basic operations: addition, subtraction, division, multiplication, exponentiation; when and how to use operations.
2. Structure: primes, factors, and multiples.
3. Number theory: modular arithmetic (including parity); Diophantine equations; Fibonacci sequence; Pascal's triangle.
4. Approximation: rounding; bounds; approximate calculation; interpolation and extrapolation; estimation.
5. Ratios: use of ratios, rates, and proportions; relation to division; golden section.

C. Measurement

1. Units: systems (English, metric, non-standard); importance of standard units.

2. Spatial: length, area, volume, perimeter, and surface area.

3. Approximate nature: exact versus approximate, i.e., counting versus measuring; calculation with approximations; margin of error; propagation of error; estimation.

4. Additivity.

D. Numerical Functions and Relations

1. Relations: order, inequalities, subset relations, additivity, infinite sets.

2. Functions: linear, quadratic, exponential; rules, patterns.

3. Equations: solution techniques (e.g., manipulation, guess-and-test); missing addend and factor; relation to construction of numbers.

4. Formulas: interpretation and evaluation; algebra as generalized arithmetic.

E. Combinatorics and Counting Techniques

1. Multiplication principle and decomposition.

2. Pigeonhole principle.

3. Systematic enumeration of cases.

F. Statistics and Probability

1. Basic quantification: counting; representation by rational numbers.

2. Derived measures: average, median, range.

3. Concepts: independence, correlation; "Law of Averages."

4. Prediction: relation to probability.

5. Data processing: collection and analysis.

6. Data presentation: graphs, charts, tables; construction and interpretation.

G. Geometry

1. Dimensionality: one, two, three, and four dimensions.

2. Rigid transformations: transformations in two and three dimensions; rotations, reflections, and translations; symmetry.

3. Tessellations: covering the plane and bounded regions; kaleidoscopes; role of symmetry; other surfaces.

4. Maps and models in scale: application of ratios.

5. Perspective: rudiments of drawing in perspective; representation of three-dimensional objects in two dimensions.

6. Geometrical objects: recognition; relations among; constructions; patterns.

7. Topological mappings and properties: invariants.

APPENDIX I.B:

Descriptions of **SQUARE ONE TV**

Programs Shown to Viewers

Descriptions of **SQUARE ONE TV** Programs Shown to Viewers

The following pages provide rundowns of the **SQUARE ONE TV** programs viewed by the viewing group. Of the six weeks, three weeks came from Season I and three weeks came from Season II. The programs were selected to minimize the repetition of individual segments over the treatment period; in particular, there is no repetition of Mathnet segments. In line with this, some minor adjustments were made in the programs, principally to avoid repetition. The programs in which changes were made are marked with asterisks.

The listing here reflects the order in which the programs were viewed. All segments in each program are listed. The entries include descriptive data from the production data base. The information provided in each entry is as follows:

Line one:

Show number (Note that the first digit indicates the season in which the program originally aired);
Item number -- the serial number of the segment in its program;
Title;
Production number -- unique to each segment;
Item format -- a three letter code:

ANI	animation
BUM	bumper
GAM	game show
LAF	live-action film
NET	Mathnet episode
PAR	continuation of a multi-part segment
SON	song
STU	studio sketch

Length -- the running time of the segment.

Line two:

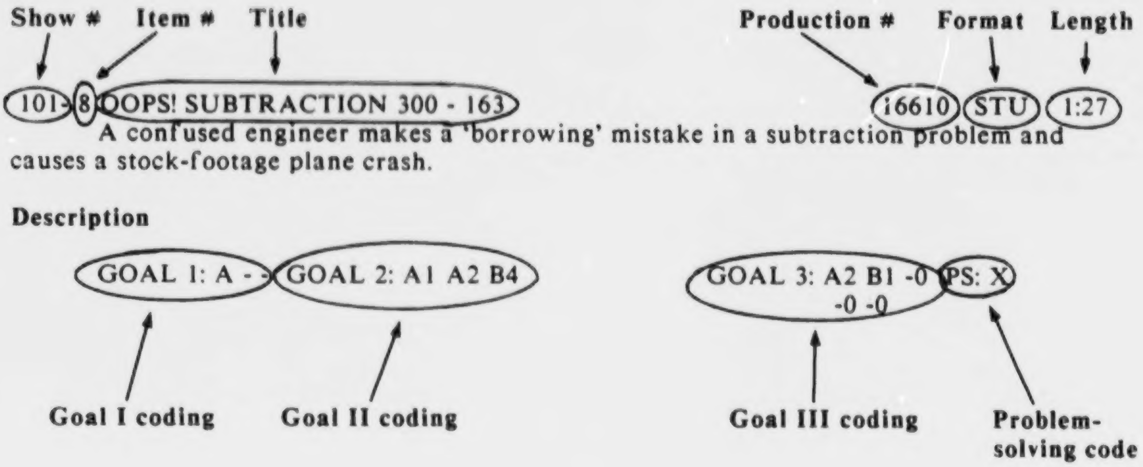
Brief description -- included for all but bumpers (a segue between segments of a program).

Last line:

Goal I classification;
Goal II classification;
Goal III classification;
Problem-solving notation -- "X" indicates that the segment is a Problem-solving segment.

(Note: "-0-" signifies either that a brief description is not necessary or that the segment does not address a particular goal. The goal content of continuations of multi-part segments (PAR) is ordinarily coded under the first part. Hence the goal classifications for segments marked "PAR" are "-0-".)

EXAMPLE:



101 8 OOPS! SUBTRACTION 300 - 163 16610 STU 1:27

A confused engineer makes a 'borrowing' mistake in a subtraction problem and causes a stock-footage plane crash.

Description

GOAL 1: A - - GOAL 2: A1 A2 B4 GOAL 3: A2 B1 -0 -0 -0 PS: X

SQUARE ONE TV RUNDOWNS

101- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

101- 2 INFINITY (SONG) 10230 SON 3:18
The song introduces the idea that there is no largest number. The graphics suggest several infinite collections to support the song.

GOAL 1: B C - GOAL 2: -0- GOAL 3: D1 B1 -0 PS: -
-0 -0

101- 3 (INFINITY) NEWSROOM INTERRUPT 16620 BUM 0:10
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

101- 4 MATHMAN: MULTIPLES OF 3 15630 ANI 1:21
Mathman plays a video game in which he must eat only multiples of 3.

GOAL 1: C - - GOAL 2: -0- GOAL 3: B2 -0 -0 PS: -
-0 -0

101- 5 PHONER: THE ANSWER IS 3 15970 STU 2:23
Arthur has a one-sided telephone conversation in which he chooses a number and performs a series of operations that always gives him the answer of 3.

GOAL 1: A C - GOAL 2: -0- GOAL 3: D2 B1 -0 PS: -
-0 -0

101- 6 INFINITY REPRISE: 1 10231 BUM 0:02
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

101- 7 BATTLE OF THE BULGE CATERERS: SANDWICHES 11460 STU 4:41
The Battle of the Bulge Catering Company must make more than 11 different sandwich combinations from two meats and three cheeses. The problem introduces the multiplication principle from combinatorics.

GOAL 1: A C - GOAL 2: A1 A2 A3 B6 D1 GOAL 3: E1 D2 -0 PS: X
C1e C2c C4a -0 -0

101- 8 OOPS! SUBTRACTION 300 - 163 16610 STU 1:27
A confused engineer makes a 'borrowing' mistake in a subtraction problem and causes a stock-footage plane crash.

GOAL 1: A - - GOAL 2: A1 A2 B4 GOAL 3: A2 B1 -0 PS: X
-0 -0

101- 9 (INFINITY: 2) NEWSROOM INTERRUPT 16621 BUM 0:02
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

101-10 (PERFECT SQUARES INTRO) LOGO 16570 BUM 0:03
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

101-11 PERFECT SQUARES 13140 SON 3:25
A blues band sings about square numbers and graphically suggests their connection to geometry.

GOAL 1: A C - GOAL 2: -0- GOAL 3: B2 B1 -0 PS: -
-0 -0

101-12 BUREAU OF MISSING NUMBERS: 14 15930 STU 1:59
Terry Ryan, an FBI type, takes information pertaining to the number 14 and inputs this information into her computer. These characteristics include factors, whether or not it is prime or square, etc.

GOAL 1: A - - GOAL 2: A1 B3 B4 C2c GOAL 3: B2 B1 -0 PS: X
-0 -0

101-13 INFINITY REPRISE:2 10232 BUM 0:02
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

101-14 MATHNET-PROBLEM OF THE MISSING MONKEY-1 11031 NET 8:05
The Mathnetters investigate a series of burglaries allegedly committed by a monkey that escaped from the zoo.

GOAL 1: C - - GOAL 2: A1 A2 B2 B3 C4a GOAL 3: C3 D1 -0 PS: X
C4b -0 -0

101-15 INFINITY REPRISE:3 10233 BUM 0:03
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

102- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

102- 2 STAR TRUCK BLANDSTAND 13130 STU 7:34
Space travellers find themselves contestants on a game show, rating songs. The song with the highest average score wins. The characters learn that an average can't be higher than the highest score.

GOAL 1: A C - GOAL 2: A1 A2 B4 D1 D2 GOAL 3: F5 F6 F2 PS: X
C1c C2a B1 B4

102- 3 (BLANDSTAND) NEWSROOM INTERRUPT:18 13131 BUM 0:12
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

102- 4 RAPPIN' JUDGE 14740 SON 2:40
A judge raps his decision that a girl on a skateboard could not have committed the crime because she could not have travelled 8 miles in 2 hours if she were only going 3 miles per hour.

GOAL 1: A C - GOAL 2: A1 B1 B3 B4 D1 GOAL 3: B5 C2 B1 PS: X
C1a -0 -0

102- 5 BLACKSTONE: DIME, PENNY, NICKEL 15537 STU 3:13
Blackstone uses a fundamental property of even and odd numbers to correctly identify which hand holds the dime and which holds the penny. His follow-up trick depends on psychology--not mathematics.

GOAL 1: - - - GOAL 2: -0- GOAL 3: B3 B1 -0 PS: -
-0 -0

102- 6 (NINES INTRO) LOGO 17530 BUM 0:08
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

102- 7 NINES 15870 SON 2:34
The cast sings a country music tune expressing the idea that the digits of any multiple of 9 always add up to 9 or a multiple of 9.

GOAL 1: B C - GOAL 2: -0- GOAL 3: B2 D2 B1 PS: -
-0 -0

102- 8 WARNING 5 (UNDERSTAND KIND OF SOLUTIONS) 17585 BUM 0:09
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

102- 9 MAP, THE 14050 LAF 1:23
An older boy and his little brother use a map scale to estimate distance and travel time.

GOAL 1: A C - GOAL 2: A1 A3 B2 B3 B4 GOAL 3: G4 C3 B1 PS: X
D1 C1a -0 -0

102-10 MATHNET-PROBLEM OF THE MISSING MONKEY-2 11032 NET 9:40
In their continued search for a missing monkey, the Mathnetters come across information presented in a circle graph and use a map and compass to estimate the approximate location of the gorilla.

GOAL 1: A C - GOAL 2: A1 B1 B2 B3 B4 GOAL 3: G4 C3 B1 PS: X
B5 C1a E1 -0

103- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

103- 2 SPADE PARADE: IN SEARCH OF YUCCA PUCK -1 15901 STU 2:45
Spade Parade takes on the case of Vanessa Van Vandervan who has hired 3 consultants to tell her the route to the Yucca Puck. She doesn't know which one tells the truth, which lies, and which does both.

GOAL 1: A C - GOAL 2: A1 A2 A3 B1 B3 GOAL 3: E3 -0 -0 PS: X
D2 C1a C1e C3b -0 -0

103- 3 MATHMAN: DECIMALS LESS THAN .5 15690 ANI 1:19
Mathman plays a video game in which he must eat only decimal fractions less than .5.

GOAL 1: C - - GOAL 2: -0- GOAL 3: A4 D1 -0 PS: -
-0 -0

103- 4 SPADE PARADE: IN SEARCH OF YUCCA PUCK -2 15902 PAR 2:32
Spade Parade solves the case by asking several questions to sort out a declared liar, a truth-teller, and a third who sometimes tells the truth and sometimes lies.

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

103- 5 LESS THAN ZERO 14150 SON 2:04

This song presents a diving, dance, skating, and hammer-throw competition to show arithmetic realizations of negative numbers.

GOAL 1: C - - GOAL 2: -0- GOAL 3: A6 D1 -0 PS: -
-0 -0

103- 6 (GAME SHOW) LEAD IN 17260 BUM 0:08
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

103- 7 BUT WHO'S COUNTING?: 1 (SEASON 1) 12111 GAM 5:48
Contestants arrange five randomly chosen digits in an attempt to form the smallest possible 5 digit number. To play, they must apply some knowledge of place value and probability.

GOAL 1: A C - GOAL 2: A1 B4 D2 C1b
C2c
GOAL 3: A2 D1 F4 PS: X
-0 -0

103- 8 DATA HEADACHE II 14312 STU 1:32
A cab driver uses a pie chart to organize his business expenses and rid himself of a data headache.

GOAL 1: A - - GOAL 2: -0- GOAL 3: F6 -0 -0 PS: -
-0 -0

103- 8 EB: PONG GAME 15180 ANI 0:19
This animation illustrates billiard geometry and shows a ball rebounding from wall to wall before finally exiting the one opening.

GOAL 1: B - - GOAL 2: -0- GOAL 3: G2 G6 -0 PS: -
-0 -0

103- 9 VO: HOW MUCH LEFT 17293 BUM 0:06
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

103-10 YOU CAN COUNT ON IT 16680 SON 1:58
This song presents various ways that math shows up in the world.

GOAL 1: A C - GOAL 2: -0- GOAL 3: C1 -0 -0 PS: -
-0 -0

103-11 WARNING 3 (REMAIN CALM) 17583 BUM 0:12
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

103-12 MATHNET-PROBLEM OF THE MISSING MONKEY-3 11033 NET 8:18
The Mathnetters continue looking for the monkey, measuring the distance between footprints and using a map to figure distance, rate, and time.

GOAL 1: A C - GOAL 2: A1 B1 B2 B3 B4 GOAL 3: G4 B5 -0 PS: X
C1a C4a -0 -0

104- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

104- 2 (TONY AND THE TOGAS INTRO) LOGO 17520 BUM 0:05
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

104- 3 TONY AND THE TOGAS 12100 SON 6:25
A Phoenician singer finds himself recording a song in Rome and learns about Roman numerals in the process.

GOAL 1: - - - GOAL 2: -0- GOAL 3: A2 -0 -0 PS: -
-0 -0

104- 4 OOPS! RULER 16780 STU 1:11
A confused character causes a great accident when he fails to line up his ruler properly.

GOAL 1: A - - GOAL 2: A1 A2 B3 D1 GOAL 3: C2 -0 -0 PS: X
-0 -0

104- 5 WRONG BUILDING 11310 STU 4:30
Frank Loyd Wrong, the architect, ignores the importance of proper scaling in planning and constructing a building.

GOAL 1: A C - GOAL 2: A1 B3 B4 D1 D4 GOAL 3: G4 C2 -0 PS: X
C1a C2a C2b C3b -0 -0

104- 6 LOGO 5 GENERIC (LUISA) 17370 BUM 0:07
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

104- 7 SHOEMAKER & ELVES 11850 STU 3:39
A shoemaker wants to sue his elves for failing to interpret the scale of a plan correctly. Instead of a 1:2 ratio, they use a 2:1 ratio.

GOAL 1: A C - GOAL 2: A1 A2 A3 B4 D1 GOAL 3: G4 B5 A3 PS: X
C1a C1e C2a C4a -0 -0

104- 8 COMIC: SHRUNKEN TOOTHBRUSH 13510 STU 2:29
A comic mistakenly believes that the entire scale of the world has been altered when he unknowingly sticks his head into his daughter's dollhouse.

GOAL 1: - - - GOAL 2: A1 A3 D1 GOAL 3: G4 B5 -0 PS: X
-0 -0

104- 9 MATHNET-PROBLEM OF THE MISSING MONKEY-4 11034 NET 8:17
The Mathnetter's recognize that, sometimes, one must look at a problem from a different point of view -- and so hypothesize that they are searching for a gorilla and a man in a monkey suit.

GOAL 1: C - - GOAL 2: A1 B1 B3 C1a GOAL 3: -0 -0 -0 PS: X
C3a C4a C4b -0 -0

105- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

105- 2 PRIME TIME PROGRAMMING MEETING 12040 STU 3:58
A group of television executives meet to discuss next season's programs -- whose titles must contain only prime numbers.

GOAL 1: C - - GOAL 2: -0- GOAL 3: B2 -0 -0 PS: -
-0 -0

105- 3 PERCENTS 15380 SON 2:25
This glitzy song expresses the relations among percents, fractions, and decimals.

GOAL 1: A C - GOAL 2: -0- GOAL 3: A5 A3 A4 PS: -
-0 -0

105- 4 SODA SHOPPE 16100 LAF 0:49
Two customers use an easy way to compute a ten percent tip, which they then round up to the nearest ten cents.

GOAL 1: A C - GOAL 2: A1 A2 B2 B4 GOAL 3: A5 A4 B4 PS: X
-0 -0

105- 5 VO: STOP COMPLAINING (ANS.) 17314 BUM 0:05
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

105- 6 TROUT ON YOUR HEAD 14010 STU 1:10
This commercial uses a horizontal bar graph to illustrate that most quacks sampled suggest putting a trout on one's head as a headache remedy.

GOAL 1: A C - GOAL 2: A1 B3 D1 C1d GOAL 3: F6 A5 -0 PS: X
-0 -0

105- 7 BUT WHO'S COUNTING?: 2 (SEASON 1) 12112 GAM 6:54
Contestants arrange six randomly chosen digits in an attempt to form two 3-digit numbers with the largest possible sum. To play, they must apply some knowledge of place value and probability.

GOAL 1: A C - GOAL 2: A1 B4 B5 D1 D2 C1b C2c GOAL 3: A2 D1 F4 PS: X
-0 -0

105- 8 (VERY NICE) NEWSROOM INTERRUPT:26 14551 BUM 0:19
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

105- 9 MATHNET-PROBLEM OF THE MISSING MONKEY-5 11035 NET 10:01
George climbs atop the Hollywood sign, and the Mathnetters successfully solve the problem of the missing monkey -- putting both the gorilla and the thief behind bars.

GOAL 1: - - - GOAL 2: A1 B1 D1 C4a GOAL 3: -0 -0 -0 PS: X
-0 -0

206- 1 SHOW OPEN 15950 BUM 0:46
** -0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

206- 2 DIRK NIBLICK: ITTY BITTY BUSINESS PT.1 20220 ANI 5:35
Dirk comes to the aid of the town merchants who are having to close business due to being swindled by a crooked accountant.

GOAL 1: A - - GOAL 2: A1 A2 B1 B3 B4 C1c C2a D1 D4 GOAL 3: A4 A5 -0 PS: X
-0 -0

206- 3 MATHMAN: FRACTIONS GREATER THAN 1 20070 ANI 1:37
Mathman plays a video game in which he must eat all fractions which are greater than 1.

GOAL 1: C - - GOAL 2: -0- GOAL 3: A3 -0 -0 PS: -
-0 -0

206- 4 DIRK NIBLICK: ITTY BITTY BUSINESS PT.2 20221 PAR 3:00
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

206- 5 CLOSE CALL # 9 (SEASON 2) 20820 GAM 6:54
Students compete against each other trying to get the closest estimate to: Peanuts in the Container, Peanut Butter Jars/Elephant, Slinkies on the Rain Slickers, and % of Audience with Beach Balls.

GOAL 1: C - - GOAL 2: A1 B2 GOAL 3: A5 C1 C2 PS: X
-0 -0

206- 6 EB: MIXED NUMBERS 4/3 20950 ANI 0:15
This short animation illustrates mixed numbers by showing the same amount of liquid in a number of different glasses.

GOAL 1: A - - GOAL 2: -0- GOAL 3: A3 D1 -0 PS: -
-0 -0

206- 7 PRIME NUMBERS 20840 SON 3:42
The Jets sing a song about prime numbers.

GOAL 1: - - - GOAL 2: A1 B4 GOAL 3: B2 B3 -0 PS: X
-0 -0

206- 8 EB: PRIME NUMBERS 21360 ANI 0:23
This short animation illustrates the prime numbers on a 100 grid.

GOAL 1: - - - GOAL 2: -0- GOAL 3: B2 -0 -0 PS: -
-0 -0

206- 9 SQUARE ONE PUZZLER: RECTANGLES 21160 ANI 0:41
A short animation puzzler: How many rectangles are in the diagram? The viewer must take into account the embedded rectangles.

GOAL 1: - - - GOAL 2: A1 B4 C1b C1e GOAL 3: G6 -0 -0 PS: X
C2c -0 -0

206-10 MATHNET-CASE OF THE GREAT CAR ROBBERY-1 20010 NET 5:43
The Mathnetters are called in to investigate the increase in the number of cars being stolen in the L.A. area. They begin by analyzing data collected from the robberies in the hope of finding clues.

GOAL 1: C - - GOAL 2: A1 B2 B3 B4 C1c GOAL 3: A5 B4 F5 PS: X
C2a C3a F6 -0

207- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

207- 2 DIRKLET: SQUARE ONE SQUARES PROMO 21200 BUM 0:44
Dirk reminds Square One TV viewers to stay tuned for another exciting challenge on Square One Squares.

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

207- 3 BLACKSTONE: MAGIC SAFARI 21080 STU 3:54
Blackstone illustrates associativity by forcing the spectator to count to a pig in a collection of animals in a circle.

GOAL 1: - - - GOAL 2: -0- GOAL 3: D2 -0 -0 PS: -
-0 -0

207- 4 SQUARE ONE SQUARES # 7 20590 GAM 8:07
Two students try to determine which cast member is giving the correct answer to the questions: Embedded Squares, Blue Hair/Sunglasses, Cube with a Corner cut off, and Dozen Dozen.

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

207- 4 SQUARE ONE SQUARES # 7 QUESTION 1 20591 SOS -0-
How many squares are in the paintbox grid?

GOAL 1: C - - GOAL 2: A1 B4 C1b C1e
C2c C4a
GOAL 3: G6 -0 -0 PS: X
-0 -0

207- 4 SQUARE ONE SQUARES # 7 QUESTION 2 20592 SOS -0-
Eight people are going to a rock concert. Six of them have blue hair, four are wearing sunglasses. What is the smallest number who have blue hair and sunglasses?

GOAL 1: C - - GOAL 2: A1 A3 B4 C1b
C1e
GOAL 3: C4 -0 -0 PS: X
-0 -0

207- 4 SQUARE ONE SQUARES # 7 QUESTION 3 20593 SOS -0-
Which of two nets can be folded to form a cube with a corner cut off?

GOAL 1: C - - GOAL 2: A1 B4 C1e D1 GOAL 3: G2 G6 -0 PS: X
-0 -0

207- 4 SQUARE ONE SQUARES # 7 QUESTION 4 20594 SOS -0-
Which is more: six dozen dozen or half a dozen dozen?

GOAL 1: C - - GOAL 2: A1 B4 C2a GOAL 3: A2 D1 -0 PS: X
-0 -0

207- 5 STICK SQUARES - 3 13953 STU 0:41
Alison Smith demonstrates toothpick square tricks to the viewing audience.

GOAL 1: C - - GOAL 2: A1 B4 D1 D2 C1e GOAL 3: G6 -0 -0 PS: X
C4a -0 -0

207- 6 MATHMAN: MULTIPLES OF 5 15670 ANI 1:09
Mathman plays a video game in which he must eat only numbers that are multiples of 5.

GOAL 1: C - - GOAL 2: -0- GOAL 3: B2 -0 -0 PS: -
-0 -0

207- 7 ME AND MY SHADOW 13660 STU 2:36
Debbie Allen discusses dimensionality by comparing her own 3-dimensionality with the 2-dimensionality of her shadow.

GOAL 1: C - - GOAL 2: -0- GOAL 3: G1 C2 -0 PS: -
-0 -0

207- 8 MATHNET-CASE OF THE GREAT CAR ROBBERY-2 20011 NET 10:39
The Mathnetters continue their investigation into the missing cars and meet Li So, a young lady whose car has been stolen. Although she saw it being towed, the L.A.P.D. has no record of taking it.

GOAL 1: - - - GOAL 2: A1 B4 C1d C2c GOAL 3: A5 B4 F2 PS: X
C3a C3b C3c C4a F5 F6

208- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

208- 2 TRIPLE PLAY # 1 20640 GAM 5:48
Two students compete against each other trying to cover the vertices of an equilateral triangle. Multiplication and Addition sentences must be created in order to cover a vertex.

GOAL 1: C - - GOAL 2: A1 B4 C1b C2c GOAL 3: B1 G6 -0 PS: X
C3a -0 -0

208- 3 DIRKLET: USE GRAPHS 21340 BUM 1:17
Dirk suggests that a good problem solving heuristic is to make a graph.

GOAL 1: A C - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

208- 4 OOPS! 1/2 + 1/3 = 2/5 20460 STU 1:37
A confused character makes a mistake when adding 1/2 + 1/3 which causes a stock-footage disaster.

GOAL 1: A - - GOAL 2: A1 A2 B4 D1 GOAL 3: B1 A3 -0 PS: X
-0 -0

208- 5 COMMON MULTIPLE MAN 11890 STU 4:21
When a couple has to figure out how many hors d'oeuvres to buy to serve either 12, 16, or 24 guest equally, they call Common Multiple Man, a super hero with a very strange super power.

GOAL 1: A C - GOAL 2: A1 A3 B1 B4 D1 GOAL 3: B2 -0 -0 PS: X
D2 D4 C1c C2c C -0 -0

208- 6 MATHMAN: PENTAGONS 20150 ANI 1:33
Mister Glitch plays a video game in which he must eat all polygons that are pentagons.

GOAL 1: C - - GOAL 2: -0- GOAL 3: G6 -0 -0 PS: -
-0 -0

208- 7 ARCHIMEDES 21130 SON 2:56
This song about Archimedes highlights some of his inventions and discoveries.

GOAL 1: A - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

208- 8 MATHNET-CASE OF THE GREAT CAR ROBBERY-3 20012 NET 9:39
20,000 cars have disappeared from L.A. during the past two months with a recovery rate of only 30 percent. The Mathnetters decide to speak to a used car dealer to learn more about the missing cars.

GOAL 1: - - - GOAL 2: A1 B1 B3 C1c GOAL 3: B5 F5 F6 PS: X
C2c C3a C3b C4a -0 -0

209- 1 SHOW OPEN 15950 BUM 0:46
** -0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

209- 2 ICE CREAM STORE: CALORIES 10130 STU 3:10
A dieting woman enters an ice cream store run by a Valley Boy who uses a bar chart and percents to compare the calories of the various frozen treats.

GOAL 1: A C - GOAL 2: A1 B3 D1 C1d GOAL 3: A5 A3 D1 PS: X
F6 -0

209- 3 TESSELLATIONS 15810 SON 3:12
A boppy beach tune illustrates the concept of tessellation as surfers cover their boards and the beach with repeating geometric shapes.

GOAL 1: B - - GOAL 2: -0- GOAL 3: G3 G6 -0 PS: -
-0 -0

209- 4 DIRKLET: DIVISIBLE BY 3 21300 ANI 1:40
Dirk explains that if the digits of any whole number are added and the sum is divisible by three, then the number started with is divisible by three.

GOAL 1: A B C GOAL 2: A1 B4 C2a GOAL 3: B1 -0 -0 PS: X
-0 -0

209- 5 PIECE OF THE PIE #11 (SEASON 2) 20450 GAM 5:48
Two teams alternate guessing the most common answers to the survey question "Name your favorite musical instrument." The team that accumulates the greater percentage wins the game.

GOAL 1: A C - GOAL 2: A1 B6 C3b GOAL 3: A5 B1 D1 PS: X
F6 -0

209- 6 EB: MIXED NUMBERS 3/2 20960 ANI 0:13
This short animation illustrates mixed numbers by showing the same amount of liquid in a number of different glasses.

GOAL 1: A - - GOAL 2: -0- GOAL 3: A3 D1 -0 PS: -
-0 -0

209- 7 DATA HEADACHE III 14310 STU 1:10
A corporate executive uses a line graph to organize her expenses and rid herself of a data headache.

GOAL 1: A - - GOAL 2: -0- GOAL 3: F6 -0 -0 PS: -
-0 -0

209- 8 MATHMAN: PERCENTAGES MORE THAN 1/2 15710 ANI 1:17
Mathman plays a video game in which he must eat only percentages that are less than 1/2.

GOAL 1: C - - GOAL 2: -0- GOAL 3: A5 D1 -0 PS: -
-0 -0

209- 9 MATHNET-CASE OF THE GREAT CAR ROBBERY-4 20013 NET 10:31
With Li So's help, the Mathnetters determine that the pattern of heavy cars being stolen is an important clue in the crimes. They hypothesize that the cars are being recycled as scrap metal.

GOAL 1: - - - GOAL 2: A1 B2 B3 B4 B5 GOAL 3: B1 B4 B5 PS: X
B6 C1c C2c C3b F6 -0

210- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

210- 2 PHONER: THE ANSWER IS 6 20520 STU 2:12
Reg has a one-sided telephone conversation in which he chooses a number and performs a series of operations that always gives him the answer of six.

GOAL 1: A C - GOAL 2: -0- GOAL 3: B1 D2 -0 PS: -
-0 -0

210- 3 DIRKLET: LOOK FOR A PATTERN 21330 ANI 1:46
Dirk suggests that a good problem solving heuristic is to look for a pattern .

GOAL 1: - - - GOAL 2: A1 C2a C3a GOAL 3: D2 -0 -0 PS: X
-0 -0

210- 4 BLACKSTONE: LIAR AND TRUTHTELLER 21030 STU 3:19
One spectator is to lie, the other to tell the truth in answering Blackstone's question. Even though he does not know which is the liar, the answer to one question reveals which holds a hidden coin.

GOAL 1: A - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

210- 5 BALONEY 14300 STU 1:43
Two crazy characters visit the International House of Baloney where they have a choice of any 2 of the 4 toppings for their sandwiches. They demonstrate and make a list to determine the possibilities.

GOAL 1: A C - GOAL 2: B3 C1b C2c GOAL 3: E1 F6 -0 PS: X
-0 -0

210- 6 MATHMAN: INEQUALITY 19-C < 5 20110 ANI 1:29
Mathman plays a video game in which he must eat all numbers which satisfy the inequality 19-C < 5.

GOAL 1: C - - GOAL 2: -0- GOAL 3: B1 D1 D4 PS: -
-0 -0

210- 7 MATHNET-CASE OF THE GREAT CAR ROBBERY-5 20014 NET 15:24
The Mathnetters set a trap for the car robbers. The robbers fall for the trap, leading the Mathnetters to the scrap metal site where they catch their culprit.

GOAL 1: - - - GOAL 2: A1 B1 B3 B4 C1d GOAL 3: B4 B5 F2 PS: X
C1e C3a F5 F6

226- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

226- 2 DIRKLET: DIVISIBLE BY 5 21290 ANI 1:18
Dirk explains that a whole number with a zero or a five in the ones place is divisible by five.

GOAL 1: - - - GOAL 2: A1 C2a D1 GOAL 3: B1 -0 -0 PS: X
-0 -0

226- 3 BLACKSTONE: TURNING THE DIE 21040 STU 3:22
Blackstone asks the spectator to rotate a carefully oriented die three times according to indicated directions, then again until the top number is 1, then once more. Now showing is 4, as predicted.

GOAL 1: - - - GOAL 2: -0- GOAL 3: B3 D2 -0 PS: -
-0 -0

226- 4 RATINGS WAR 14870 STU 2:25
Larry uses a double bar graph to contrast the number of people who eat rutabagas with the number of people who watch Square One TV.

GOAL 1: A C - GOAL 2: -0- GOAL 3: F6 F5 -0 PS: -
-0 -0

226- 5 PIECE OF THE PIE # 7 (SEASON 2) 20410 GAM 6:10
Two teams alternate guessing the most common answers to the survey question "Name something you see at a parade." The team that accumulates the greater percentage wins the game.

GOAL 1: A C - GOAL 2: A1 B6 C3b D2 GOAL 3: A5 B1 D1 PS: X
F6 -0

226- 6 DRAW A MAP 16690 SON 2:11
In order for Luisa to reach Arthur's house, he gives her instructions to make a map. He includes significant landmarks and uses a scale where 1 inch stands for 1 mile.

GOAL 1: A C - GOAL 2: A1 B1 B3 C1a GOAL 3: G4 G4 C2 PS: X
-0 -0

226- 7 MATHNET-CASE OF THE MAP WITH A GAP-1 20000 NET 11:47
A boy named Bronco has asked the Mathnetters for help in solving a problem. He has a treasure map which he was able to decode using reflection with a mirror. The Mathnetters agree to help him.

GOAL 1: A C - GOAL 2: A1 A2 B2 B3 B4 GOAL 3: A4 B4 E3 PS: X
B6 C1a C1e C3b G2 G5

227- 1 SHOW OPEN 15950 BUM 0:46
* -0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

227- 2 ESTIMATION 21390 SON 3:51
A song about estimation which suggests that estimating is a quick and easy way to get an answer fast. It is quite a useful tool when an answer doesn't need to be exact.

GOAL 1: - - - GOAL 2: A1 B2 GOAL 3: C3 -0 -0 PS: X
-0 -0

227- 3 SQUARE ONE SQUARES # 1 20530 GAM 6:24
Two students try to determine which cast member is giving the correct answer to the questions: Pickle Pies, Silver Dollar, and Tables.

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

227- 3 SQUARE ONE SQUARES # 1 QUESTION 1 20531 SOS -0-
Pickle Pies of the same size are illustrated via paintbox. One pie is cut into fifths and the other into sixths. The animation shows that the pie cut into fifths will have the bigger slices.

GOAL 1: C - - GOAL 2: A1 B4 C1b C1e GOAL 3: A3 D1 -0 PS: X
-0 -0

227- 3 SQUARE ONE SQUARES # 1 QUESTION 2 20532 SOS -0-
Probability of a fair coin landing on heads is one half. The probability is independent of the number of times the coin is flipped.

GOAL 1: C - - GOAL 2: A1 B5 C3c D4 GOAL 3: F1 F3 -0 PS: X
-0 -0

227- 3 SQUARE ONE SQUARES # 1 QUESTION 3 20533 SOS -0-
Four people can sit at each of three tables when the tables are separated. When the tables are pushed together some of the space for seats is lost allowing room for only eight people.

GOAL 1: C - - GOAL 2: A1 B4 C1e GOAL 3: B1 G6 -0 PS: X
-0 -0

227- 4 GREMPOD AND BLOTMO: SPONGE CANDY 14420 STU 2:18

Grempod, a Rigelian alien, offers his pal Blotmo a ripe sea fig from the planet Xerkne if he can guess which of his 4 hands holds the treat. The probability is 1/4 that Blotmo will choose correctly.

GOAL 1: C - - GOAL 2: A1 A2 B1 B5 D1 GOAL 3: F1 A3 -0 PS: X
C2a -0 -0

227- 5 HARRY'S HAMBURGER HAVEN 14240 STU 2:27

As the characters attempt to shoot a commercial for Harry's Hamburger Haven, they note the equivalence of decimal, fraction, and percent.

GOAL 1: C - - GOAL 2: -0- GOAL 3: A4 A5 A3 PS: -
-0 -0

227- 6 EB: SPOT THE PENTAGONS 20910 ANI 0:25

This short animation asks the viewer to identify which of the polygons are pentagons.

GOAL 1: - - - GOAL 2: A1 D1 GOAL 3: G6 -0 -0 PS: X
-0 -0

227- 7 MATHNET-CASE OF THE MAP WITH A GAP-2 20001 NET 10:04

The Mathnetters join forces with Bronco and ride horses into Mulch Gulch, a deserted ghost town. They are in search of the buried treasure.

GOAL 1: - - - GOAL 2: B1 GOAL 3: -0 -0 -0 PS: -
-0 -0

228- 1 SHOW OPEN 15950 BUM 0:46

* -0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

228- 2 PHONER: THE ANSWER IS 2 20500 STU 2:50

Beverly has a one-sided telephone conversation in which she chooses a number and performs a series of operations that always give her the answer of two.

GOAL 1: A C - GOAL 2: -0- GOAL 3: B1 D2 -0 PS: -
-0 -0

228- 3 BLACKSTONE: 1089 10372 STU 3:14

Blackstone asks the spectator to take a 3 digit number, reverse the digits, subtract the smaller from the larger, reverse those digits (treat it as a 3-digit number), and gets the answer 1089.

GOAL 1: - - - GOAL 2: -0- GOAL 3: D2 G2 B1 PS: -
-0 -0

228- 4 TRIPLE PLAY # 8 20710 GAM 5:24
Two students compete against each other trying to cover the vertices of an equilateral triangle. Multiplication and Addition sentences must be created in order to cover a vertex.

GOAL 1: C - - GOAL 2: A1 A3 B4 C1b GOAL 3: B1 G6 -0 PS: X
C2c C3a -0 -0

228- 5 AVERAGE AMERICAN 10220 SON 3:02
In this song, Larry sings about the statistical averages for various American habits to show Cynthia just how much of an "Average American" he is.

GOAL 1: C - - GOAL 2: -0- GOAL 3: F2 -0 -0 PS: -
-0 -0

228- 6 MATHNET-CASE OF THE MAP WITH A GAP-3 20002 NET 12:17
The Mathnetters and Bronco use triangulation to help locate the buried treasure. Their digging proves successful, not in finding the treasure, but in locating the other part of the map.

GOAL 1: A C - GOAL 2: A1 A2 B1 B2 B3 GOAL 3: B5 C1 C2 PS: X
B4 B5 C1a C1e C G4 G6

229- 1 SHOW OPEN 15950 BUM 0:46
*** -0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

229- 2 CLOSE CALL # 8 (SEASON 2) 20810 GAM 8:14
Students compete against each other trying to get the closest estimate to: Puzzle Pieces, Elephant with a Tutu, Candies on the Close Call Sign, and % of Audience with Baseball Caps.

GOAL 1: C - - GOAL 2: A1 B2 C2 GOAL 3: A5 C1 C2 PS: X
-0 -0

229- 3 MATHMAN: HEXAGONS 20140 ANI 1:21
Mathman plays a video game in which he must eat all polygons which are hexagons.

GOAL 1: C - - GOAL 2: -0- GOAL 3: G6 -0 -0 PS: -
-0 -0

229- 4 DIRK NIBLICK: MALL OR NOTHING AT ML PT.1 20260 ANI 6:07
Dirk comes to the rescue of Fluff and Fold who are being deceived by a biased survey.

GOAL 1: B - - GOAL 2: A1 A2 B1 B3 C1c GOAL 3: A5 F5 -0 PS: X
C2a D1 D4 -0 -0

229- 5 FIVE-NINETEEN BLUES 16170 LAF 1:18
This song shows that you can round off a lot of numbers but not the time the train leaves.

GOAL 1: A C - GOAL 2: -0- GOAL 3: B4 -0 -0 PS: -
-0 -0

229- 6 DIRK NIBLICK: MALL OR NOTHING AT ML PT.2 20261 PAR 2:38
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

229- 7 MATHNET-CASE OF THE MAP WITH A GAP-4 20003 NET 8:08
The Mathnetters and Bronco search for information which will help them decode their newly found map. After much trial and error, they realize that a mirror is the solution to deciphering the map.

GOAL 1: C - - GOAL 2: A1 A2 A3 B2 B4 GOAL 3: G2 -0 -0 PS: X
B6 C3a C3b C3c -0 -0

230- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

230- 2 TIME KEEPER 21410 SON 3:31
Tempest Bledsoe sings about keeping time in a factory and the clock arithmetic which is involved.

GOAL 1: C - - GOAL 2: A1 B4 C1b GOAL 3: B3 -0 -0 PS: X
-0 -0

230- 3 DIRK NIBLICK: THE LINT TRAP PT.1 20270 ANI 4:55
Dirk helps Fluff and Fold understand that they were underpaid by their boss, Soapy LaFong. He paid them for only four hours a day although they had clearly worked four and a half hours each day.

GOAL 1: B - - GOAL 2: A1 B4 C1c C2a GOAL 3: B3 -0 -0 PS: X
C2b -0 -0

230- 4 EB: ROTATIONAL SYMMETRY #2 16440 ANI 0:31
This short animation uses a 5-point star to illustrate the concept of rotational symmetry.

GOAL 1: B - - GOAL 2: -0- GOAL 3: G2 -0 -0 PS: -
-0 -0

230- 5 MATHMAN: SYMMETRY 20180 ANI 1:36
Mathman plays a video game in which he must eat all polygons which have a line of symmetry.

GOAL 1: C - - GOAL 2: -0- GOAL 3: G2 -0 -0 PS: -
-0 -0

230- 6 DIRK NIBLICK: THE LINT TRAP PT.2 20271 PAR 2:48
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

230- 7 EB: DECIMALS/PERCENTS/FRACTIONS-25% 17030 ANI 0:25
This short animation uses a square to illustrate the equivalence of 25%, .25, 25/100, and 1/4.

GOAL 1: B - - GOAL 2: -0- GOAL 3: A5 A3 A4 PS: -
-0 -0

230- 8 DIET LITE WET 14230 STU 3:22
As the characters attempt to shoot a commercial for Diet Lite Wet, they note the equivalence of fraction, decimal,and percent.

GOAL 1: A C - GOAL 2: -0- GOAL 3: A3 A5 A4 PS: -
-0 -0

230- 9 SQUARE ONE PUZZLER: CALENDAR 21140 ANI 0:58
A short animation puzzler: If today is Wednesday, what day of the week will it be in twenty days?

GOAL 1: - - - GOAL 2: A1 B4 C1b C1e GOAL 3: B3 -0 -0 PS: X
C2a C2c -0 -0

230-10 MATHNET-CASE OF THE MAP WITH A GAP-5 20004 NET 7:58
The Mathnetters help Bronco find the buried treasure after combining the two map pieces and using triangulation to locate the burial spot.

GOAL 1: A C - GOAL 2: A1 B1 B3 B4 B6 GOAL 3: C1 C2 G4 PS: X
C1a C1e C4b G6 -0

111- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

111- 2 NEWSROOM INTERRUPT: 7 (PROBAB. COIN TOSS 16910 BUM 0:21
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: F4 -0 -0 PS: -
-0 -0

111- 3 CABOT & MARSHMALLOW: PROBABILITY - 1 15471 STU 1:41
Cabot and Marshmallow discuss the meaning of a probability of one as Marshmallow creates a situation where any outcome results in a win for Cabot.

GOAL 1: C - - GOAL 2: A1 B1 B5 D1 C2a GOAL 3: F1 D1 -0 PS: X
-0 -0

111- 4 ON THE MIDWAY 14320 STU 1:42
Freddy Koehler exposes the unequal probability of winning a game in which 3/4 of the spinner is one color and 1/4 of the spinner is another color.

GOAL 1: A C - GOAL 2: A1 B1 D1 C1e GOAL 3: F4 F1 A3 PS: X
-0 -0

111- 5 (C & M: PROBABILITY INTRO) WARNING 17610 BUM 0:06
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

111- 6 CABOT & MARSHMALLOW: PROBABILITY - 2 15472 STU 2:19
Cabot and Marshmallow discuss the meaning of a probability of zero after Cabot tricks Marshmallow by playing the shell and the pea game -- without the pea.

GOAL 1: A C - GOAL 2: A1 A3 B5 D1 C1e GOAL 3: F1 -0 -0 PS: X
-0 -0

111- 7 GHOST OF A CHANCE 11950 SON 4:20
At a haunted house, a pizza delivery boy finds himself in several threatening situations -- each of which has a different probability of escape.

GOAL 1: A C - GOAL 2: A1 A2 B5 B6 GOAL 3: F1 F3 -0 PS: X
-0 -0

111- 8 LOGO 13 GENERIC (BEV) 17450 BUM 0:08
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

111- 9 HANDSHAKE CONTEST 12551 STU 4:27
The Wide Wide World of Sports You Never Heard Of covers a Handshake Competition where the newscasters learn that the number of handshakes will always be a triangular number.

GOAL 1: A C - GOAL 2: A1 A3 B4 D3 C1b GOAL 3: B2 D2 D1 PS: X
C2c C3a -0 -0

111-10 BLACKSTONE: MENTAL SPELLER 13442 STU 2:11
Blackstone asks the cast to choose an object on the table and uses the number of letters in its name as a clue to identifying it.

GOAL 1: - - - GOAL 2: -0- GOAL 3: D2 -0 -0 PS: -
-0 -0

111-11 MATHNET-PROBLEM OF THE PASSING PARADE-1 11011 NET 9:51
In anticipation of a rock star's visit, the Mathnetters calculate how much time a parade will take, estimate crowd size, and approximate the number of officers needed for crowd control.

GOAL 1: A - - GOAL 2: A1 A2 B2 B3 B4 GOAL 3: B4 B1 B5 PS: X
B6 D1 C1a C2c G4 -0

112- 1 SHOW OPEN 15950 BUM 0:46

-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

112- 2 (PAPER RACE INTRO) LOGO 17480 BUM 0:03
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

112- 3 PAPER RACE 12600 STU 5:38
Ned Flimsy visits his law professor who has him round up the costs of his books to estimate whether he has enough money.

GOAL 1: A C - GOAL 2: A1 B2 B3 B4 D1 GOAL 3: B4 B1 A4 PS: X
C1b C2b C2c -0 -0

112- 4 BURGER PATTERN 12140 SON 3:16
The Fat Boys use hamburgers to illustrate a triangular number pattern.

GOAL 1: A - - GOAL 2: A1 B4 D1 C1b GOAL 3: D2 D1 B1 PS: X
C3a -0 -0

112- 5 BUT WHO'S COUNTING?: 6 (SEASON 1) 12116 GAM 5:48
Contestants arrange 5 randomly chosen digits to create a 3 digit plus 2 digit addition problem. The largest sum wins. Contestants must apply some knowledge of place value and probability to play.

GOAL 1: A C - GOAL 2: A1 B4 D2 C1b GOAL 3: A2 D1 F4 PS: X
C2c -0 -0

112- 6 (PALINDROME INTRO) LOGO 17620 BUM 0:12
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

112- 7 IT'S A PALINDROME 14110 SON 2:53
A tango dance serves as the backdrop for a song about the definition and generation of palindromes -- numbers that read the same backwards and forwards.

GOAL 1: C - - GOAL 2: -0- GOAL 3: A2 -0 -0 PS: -
-0 -0

112- 8 MATHNET-PROBLEM OF THE PASSING PARADE-2 11012 NET 9:07
In their attempt to find a kidnapped rock star, the Mathnetters tip a bottle with liquid in it to recreate a mountain's angle of incline. They also use musical beats to keep track of time.

GOAL 1: A C - GOAL 2: A1 B1 B2 B3 B6 GOAL 3: C3 G6 G4 PS: X
C1e C2c -0 -0

113- 1 SHOW OPEN 15950 BUM 0:46
***** -0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

113- 2 GROANING WALL I 11871 STU 1:15
The cast tells each other riddles -- all of which have a mathematical theme.

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

113- 3 SAMURAI MATHEMATICIAN 14490 STU 3:11
A samurai warrior breaks similar sized boards into different numbers of equal-sized pieces to prove that the more fractional pieces you divide an object into, the smaller those pieces will be.

GOAL 1: C - - GOAL 2: A1 B4 D4 C1e GOAL 3: D1 A3 -0 PS: X
C2a C2c C3a -0 -0

113- 4 MATHMAN: EQUIVALENT FRACTIONS (1/3) 15650 ANI 1:24
Mathman plays a video game in which he must eat only fractions equivalent to 1/3.

GOAL 1: C - - GOAL 2: -0- GOAL 3: A3 -0 -0 PS: -
-0 -0

113- 5 ACTION AT THE FRACTION BAR 13250 SON 2:24
This music video takes place at the futuristic Fraction Bar and uses vocabulary words associated with fractions. It also mentions the relation between fractions, decimals, and percents.

GOAL 1: - - - GOAL 2: -0- GOAL 3: A3 A4 A5 PS: -
-0 -0

113- 6 (FRACTIONAL BASEBALL) LEAD IN 17490 BUM 0:05
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

113- 7 GOOD SPORTS: FRACTIONAL BASEBALL 10020 STU 3:45
Good Sports visits a Fractional Baseball game which is played much like regular baseball -- except that a player receives 1/4 each time he gets on base. One adds the fractions to compute the score.

GOAL 1: C - - GOAL 2: -0- GOAL 3: A3 D1 B1 PS: -
-0 -0

113- 8 VO: STOP COMPLAINING 17312 BUM 0:07
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

113- 9 OOPS! DIVISION 6 INTO 4212 16750 STU 1:50
A confused scientist fails to acknowledge place value and makes a long division mistake that causes a stock-footage disaster.

GOAL 1: A - - GOAL 2: A1 A2 B4 D1 GOAL 3: A2 B1 -0 PS: X
-0 -0

113-10 MATHNET-PROBLEM OF THE PASSING PARADE-3 11013 NET 10:06
As they gather clues to the kidnapping case, the Mathnetters attempt to decode a message, use a car registration database, and measure the width and tread of a car tire.

GOAL 1: A C - GOAL 2: A1 A2 B1 B3 C1b GOAL 3: C2 -0 -0 PS: X
C2c C3a C4b -0 -0

114- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

114- 2 (C & M: WHAT IS A NAME) LOGO 17500 BUM 0:05
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

114- 3 CABOT & MARSHMALLOW: WHAT IS A NAME 14170 STU 3:33
Cabot and Marshmallow carry on a 'Who's On First' conversation and combine logic with rates to calculate how long it will take Marshmallow's blind date to arrive.

GOAL 1: A C - GOAL 2: A1 A2 B1 B4 D1 GOAL 3: B4 B5 B1 PS: X
C2c C3c -0 -0

114- 4 (WHITHER WEATHER) NEWSROOM INTERRUPT:16 13121 BUM 0:08
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

114- 5 WHITHER WEATHER 13120 STU 4:27
A television meteorologist meaures the snowfall in 6 different areas before calculating an average snowfall.

GOAL 1: A C - GOAL 2: A1 B3 B4 D1 C1d GOAL 3: F2 B1 D1 PS: X
C1e C2a -0 -0

114- 6 WARNING 1 (ASK A FRIEND) 17581 BUM 0:10
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

114- 7 SQUARE SONG 13620 SON 2:30
This song deals with the geometrical properties of squares. Computer graphics aid greatly in illustrating the geometry of a square.

GOAL 1: A B C GOAL 2: -0- GOAL 3: G6 -0 -0 PS: -
-0 -0

114- 8 STATION PROMO (LARRY) 14265 BUM 0:04
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

114- 9 PERSON ON THE STREET: RHOMBUS 13003 LAF 1:03
The Person On the Street interviewer asks a variety of people what a rhombus is.

GOAL 1: C - - GOAL 2: -0- GOAL 3: G6 -0 -0 PS: -
-0 -0

114-10 BLACKSTONE: THE IMAGINATION DICE 15530 STU 3:14

Blackstone performs a number trick that works for any number less than 10: double it, add 2, multiply it by 5, subtract the original number, add the digits -- the answer will always be 10.

GOAL 1: - - - GOAL 2: -0- GOAL 3: D2 B3 B1 PS: -
-0 -0

114-11 DANCE OF THE GEO SHAPES:CUBE 13601 ANI 0:21

Computer graphics illustrate and highlight a cube as it rotates around itself.

GOAL 1: B - - GOAL 2: -0- GOAL 3: G6 G1 -0 PS: -
-0 -0

114-12 IN SEARCH OF THE GIANT SQUID 13480 STU 3:51

The navigator of a submarine fails to consider the concept of scale -- and mistakenly thinks that they are only centimeters away from a giant iceberg.

GOAL 1: A C - GOAL 2: A1 B3 B4 D1 D4 GOAL 3: C1 G4 -0 PS: X
C1a -0 -0

114-13 MATHNET-PROBLEM OF THE PASSING PARADE-4 11014 NET 7:21

In trying to decode Stringbean's musical message, the Mathnetters recognize that each note of the message corresponds to a tone/number on a touch-tone phone.

GOAL 1: A C - GOAL 2: A1 A2 B1 B3 C4a GOAL 3: -0 -0 -0 PS: X
C4b -0 -0

115- 1 SHOW OPEN 15950 BUM 0:46

-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

115- 2 (DADDY KNOWS DIFFERENT) LEAD IN 17280 BUM 0:05

-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

115- 3 DADDY KNOWS DIFFERENT: STAINLESS FORKS 12990 STU 5:06

Rusty gives his father the option of paying him a fixed sum or starting with a penny, doubling the previous day's amount for a month. The amount on the 30th day would be well over $5 million.

GOAL 1: A C - GOAL 2: A1 A2 B4 D1 D4 GOAL 3: B1 A1 D2 PS: X
C1c C4a -0 -0

115- 4 (PROBLEM SONG INTRO) LOGO 17510 BUM 0:05
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

115- 5 PROBLEM SONG 10190 SON 2:28
Arthur solves the problem of how many apples he and another character can peel in 3 hours by using addition, multiplication, and division.

GOAL 1: A C - GOAL 2: A1 B4 C1c C2a C2c GOAL 3: B5 A3 B1 PS: X
-0 -0

115- 6 TESSELLATION ANIMATION:TILE 10740 ANI 1:13
This animation shows both hexagons tessellating alone and hexagons forming a tessellated pattern with a star shape. The final image shows the same tessellated pattern on a real-life tile mosaic.

GOAL 1: A B - GOAL 2: -0- GOAL 3: G3 G6 -0 PS: -
-0 -0

115- 7 BUT WHO'S COUNTING?: 7 (SEASON 1) 12117 GAM 6:41
Contestants arrange five randomly chosen digits to set up a 3 digit plus 2 digit addition problem. The largest sum wins. Contestants must apply some knowledge of place value and probability to play.

GOAL 1: A C - GOAL 2: A1 B4 D2 C1b C2c GOAL 3: A2 D1 F4 PS: X
-0 -0

115- 8 DATA HEADACHE I 14311 STU 1:10
A woman uses a bar chart to organize her monthly expenses and rid herself of a data headache.

GOAL 1: A - - GOAL 2: -0- GOAL 3: F6 -0 -0 PS: -
-0 -0

115- 9 MATHNET-PROBLEM OF THE PASSING PARADE-5 11015 NET 9:01
The Mathnetters successfully solve the problem and rescue Steve Stringbean.

GOAL 1: A B - GOAL 2: A1 B1 B3 C2c GOAL 3: D2 F4 -0 PS: X
-0 -0

106- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

106- 2 ROBIN HOOD 11640 STU 6:08
Robin Hood competes in an archery contest that allows 6 arrows to get the highest odd score. With a target containing only odd zones, Robin has his last arrow miss the target to ensure an odd score.

GOAL 1: A C - GOAL 2: A1 A2 A3 B4 D1 GOAL 3: B3 B1 -0 PS: X
C1e C2a C2c C4a -0 -0

106- 3 CABOT & MARSHMALLOW: HEY CABOT 14090 STU 1:45
Cabot offers to pay Marshmallow nothing for 5 days to repay a 5 dollar bet before Marshamllow recognizes that zero times any number is zero.

GOAL 1: - - - GOAL 2: A1 B3 B4 D1 D2 GOAL 3: B1 -0 -0 PS: X
C2a -0 -0

106- 4 PERSON ON THE STREET: DODECAHEDRON 13002 LAF 0:47
The Person on the Street Interviewer asks a variety of people what a dodecahedron is.

GOAL 1: C - - GOAL 2: -0- GOAL 3: G6 -0 -0 PS: -
-0 -0

106- 5 DANCE OF THE GEO SHAPES:DODECAHEDRON 13606 ANI 0:23
Computer graphics illustrate and highlight a dodecahedron as it rotates in space.

GOAL 1: B - - GOAL 2: -0- GOAL 3: G6 G1 -0 PS: -
-0 -0

106- 6 (BUT WHO'S COUNTING? T.S.) LOGO 17540 BUM 0:07
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

106- 7 BUT WHO'S COUNTING? - TEST SHOW 10150 GAM 5:17
Contestants arrange five randomly chosen digits in an attempt to form the largest possible 5-digit number. To play, they must apply some knowledge of place value and probability.

GOAL 1: A C - GOAL 2: A1 B4 D2 C1b GOAL 3: A2 D1 F4 PS: X
C2c -0 -0

106- 8 DISCLAIMER: ANGLES 10911 BUM 0:08
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: G6 -0 -0 PS: -
-0 -0

106- 9 ANGLE DANCE 10180 SON 2:23
The rock group Plane Geometry sings a song about angles and uses body movement to illustrate angles, as well.

GOAL 1: B C - GOAL 2: -0- GOAL 3: G6 -0 -0 PS: -
-0 -0

106-10 PLAYING THE ANGLE 15330 LAF 3:00

Nancy Lieberman, the first woman to play professional basketball, talks about and demonstrates the mathematics involved in basketball. She cites angles and parabolas, in particular.

GOAL 1: A C - GOAL 2: A1 A2 A3 B2 B5 GOAL 3: G6 F4 C2 PS: X
A3 A5

106-11 EB: PONG GAME 15180 ANI 0:19

This animation illustrates billiard geometry and shows a ball rebounding from wall to wall before finally exiting the one opening.

GOAL 1: B - - GOAL 2: -0- GOAL 3: G2 G6 -0 PS: -
-0 -0

106-12 MATHNET-CASE OF THE MISSING BASEBALL-1 10540 NET 6:27

The Mathnetters investigate a missing baseball by determining the angle at which it would have rebounded off a billboard.

GOAL 1: A - - GOAL 2: A1 B1 B3 B4 C1a GOAL 3: G6 G4 -0 PS: X
C2c -0 -0

107- 1 SHOW OPEN 15950 BUM 0:46

-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

107- 2 IDENTITY CRISIS 10090 STU 4:38

The number Zero visits a psychiatrist who cures his identity crisis by stressing the role zero plays in place value and multiplication.

GOAL 1: C - - GOAL 2: -0- GOAL 3: A2 A4 B1 PS: -
D1 -0

107- 3 (LEMONADE STAND INTRO) LOGO 17460 BUM 0:07

-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

107- 4 LEMONADE STAND IN THE DESERT 14340 STU 2:46

Shari Belafonte Harper runs a lemonade stand that sells lemonade for 26% of one dollar. She and Arthur discuss percent and decimal relations -- especially as they pertain to money.

GOAL 1: A C - GOAL 2: A1 B4 C2a GOAL 3: A5 D1 A3 PS: X
-0 -0

107- 5 EIGHT PERCENT OF MY LOVE 11480 SON 2:47
Cris uses percentages to sing about the various ways his love is divided. As Cris mentions a percentage, a drummer displays the corresponding wedge of a pie chart.

GOAL 1: A C - GOAL 2: -0- GOAL 3: A5 F6 -0 PS: -
-0 -0

107- 6 MATINEE MOVIE PROMO 12631 BUM 0:21
Shirly Schlemmer asks her viewing audience to tune into "Cartablanca", today's Matinee Movie.

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

107- 7 VO: DON'T DESPAIR 17325 BUM 0:06
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

107- 8 HARRY'S HAMBURGER HAVEN 14240 STU 2:27
As the characters attempt to shoot a commercial for Harry's Hamburger Haven, they note the equivalence of decimal, fraction, and percent.

GOAL 1: C - - GOAL 2: -0- GOAL 3: A4 A5 A3 PS: -
-0 -0

107- 9 MATINEE MOVIE: CARTABLANCA 12630 STU 6:20
As three characters attempt to leave Cartablanca by plane, an Inspector rounds up their weights to make sure that they do not exceed the maximum load.

GOAL 1: A - - GOAL 2: A1 A2 B2 B4 D1 GOAL 3: B4 B1 -0 PS: X
-0 -0

107-10 YES, GENERAL, SIR 12960 STU 1:52
A private demonstrates the six different was one can order the three words 'yes', 'general', and 'sir.' She also demonstrates this visually by arranging 3 fruits -- apple, pear, and orange.

GOAL 1: C - - GOAL 2: A1 B4 D1 D4 C1e GOAL 3: E1 -0 -0 PS: X
-0 -0

107-11 MATHNET-CASE OF THE MISSING BASEBALL-2 10630 NET 5:23
The Mathnetters gather facts and use logical reasoning to determine the whereabouts of a missing house.

GOAL 1: A - - GOAL 2: A1 B1 B3 C1a GOAL 3: G4 -0 -0 PS: X
C1e C4a C4b -0 -0

108- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

108- 2 SUDS: POPCORN 10080 STU 3:38
Arthur learns that doubling the length, width, and depth of a box increases its volume eight times.

GOAL 1: A C - GOAL 2: A1 B1 B4 D1 D3 GOAL 3: C2 C1 D2 PS: X
C1e C2a C4a -0 -0

108- 3 OOPS! DECIMALS 16770 STU 1:38
A confused doctor forgets to line up the decimals in a 3-digit addition problem and causes a stock-footage disaster.

GOAL 1: A - - GOAL 2: A1 A2 B4 D1 GOAL 3: A4 B1 PS: X
-0 -0

108- 4 LOGO 6 GENERIC (LUISA) 17380 BUM 0:06
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

108- 5 CABOT & MARSHMALLOW: WOODEN CANDY BARS 14820 STU 2:20
Cabot uses 3 differently shaped rectangular wooden blocks to illustrate that objects with different dimensions can still have the same volume.

GOAL 1: A C - GOAL 2: A1 A2 B2 B3 B4 GOAL 3: C2 C1 -0 PS: X
C1e C2a C4a -0 -0

108- 6 COUNTRY AND WESTERN MUSIC PITCH 15450 STU 1:56
Two country and western singers recount the titles of their greatest hits, all of which make mention of relations involving fractions.

GOAL 1: - - - GOAL 2: -0- GOAL 3: A3 D1 -0 PS: -
-0 -0

108- 7 TESSELLATIONS 15810 SON 3:12
A boppy beach tune illustrates the concept of tessellation as surfers cover their boards and the beach with repeating geometric shapes.

GOAL 1: B - - GOAL 2: -0- GOAL 3: G3 G6 -0 PS: -
-0 -0

108- 8 BUT WHO'S MULTIPLYING?: 12 16941 GAM 5:33
Two contestants attempt to cover three numbers in a row by selecting factors of these numbers from the Factor Board and calling out the resultant product.

GOAL 1: A C - GOAL 2: A1 B4 B6 C1b GOAL 3: B1 B2 D1 PS: X
C1c C2c F4 -0

108- 9 MATHMAN: FACTORS OF 18 15570 ANI 1:12
Mathman plays a video game in which he must eat all numbers that are factors of 18.

GOAL 1: C - - GOAL 2: -0- GOAL 3: B2 -0 -0 PS: -
-0 -0

108-10 GROANING WALL II 11872 STU 1:09
The cast tells each other riddles, all of which have a mathematical theme.

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

108-11 MATHNET-CASE OF THE MISSING BASEBALL-3 10670 NET 6:17
The Mathnetters continue their search for the missing house, using a database to access information about a pair of glasses that have turned up on the property.

GOAL 1: A - - GOAL 2: A1 B1 B3 C1a GOAL 3: F4 D2 -0 PS: X
C2c C3a -0 -0

109- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

109- 2 DISCLAIMER: ODD NUMBERS 17600 BUM 0:08
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: B2 -0 -0 PS: -
-0 -0

109- 3 PHONEYMOONERS: HOLE IN THE WALL 13930 STU 7:00
Alph and Throckmorton must estimate how many bricks they need to repair a hole in the wall caused by Alph's bowling ball. They draw a diagram to figure out the area of this irregular shape.

GOAL 1: A C - GOAL 2: A1 A2 A3 B4 D1 GOAL 3: C2 C1 G4 PS: X
C1b C4a B1 -0

109- 4 NEWSROOM INTERRUPT:24(AREA IRREG.SHAPE) 16920 BUM 0:11
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

109- 5 NAVIGATOR 15320 LAF 3:47
A female navigator in the New York harbor talks about how she uses maps to chart her course.

GOAL 1: A - - GOAL 2: A1 A2 B3 B4 C1a GOAL 3: C2 G4 G6 PS: X
-0 -0

109- 6 BLACKSTONE: HEADS OR TAILS 15535 STU 3:03
After a spectator has turned over pairs of coins from a pile of 10 dimes, Blackstone uses the principle of parity to correctly determine whether a covered coin is heads or tails.

GOAL 1: - - - GOAL 2: -0- GOAL 3: B3 -0 -0 PS: -
-0 -0

109- 7 X...IT'S THE SIGN OF THE TIMES 13580 SON 3:33
The cast gives a Hispanic flavor to this song about the multiplication symbol.

GOAL 1: A C - GOAL 2: -0- GOAL 3: B1 -0 -0 PS: -
-0 -0

109- 8 MATHMAN: ODD NUMBERS 15580 ANI 1:12
Mathman plays a video game in which he must eat only odd numbers.

GOAL 1: C - - GOAL 2: -0- GOAL 3: B3 -0 -0 PS: -
-0 -0

109- 9 MATHNET-CASE OF THE MISSING BASEBALL-4 10710 NET 7:42
The Mathnetters determine the worth of stolen gold bars as they piece together a picture of the man who may have stolen the house. They also use a map to determine the range a helicopter could fly.

GOAL 1: A B - GOAL 2: A1 B1 B2 B3 B4 GOAL 3: B4 G4 C3 PS: X
B5 C1a C2c C3a B5 -0

110- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

110- 2 SUPERGUY STANDARD OPEN 11420 BUM 0:37
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

110- 3 SUPERGUY: NEW CAPE CAPER - 1 11431 STU 3:10
Superguy wonders how many outfits he can make from 3 belts and 3 capes. Using combinatorics, he develops a systematic way of counting and combining results.

GOAL 1: A C - GOAL 2: A1 A3 B4 D3 C1e GOAL 3: E1 D2 B1 PS: X
C3a C3b -0 -0

110- 4 CALVIN KLEIN BOY 16140 LAF 1:22
Dweezil Zappa discovers the meaning of combinatorics when he determines how many possible outfits he can make from a certain number of pants, shirts, and sweaters.

GOAL 1: C - - GOAL 2: -0- GOAL 3: E1 -0 -0 PS: -
-0 -0

110- 5 SUPERGUY: NEW CAPE CAPER - 2 11432 PAR 3:40
Using combinatorics, Superguy lays out all the possible combinations for 3 belts and 3 capes. He gets 9 outfits and recognizes that he could have multiplied the numbers of capes and belts instead.

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

110- 6 VO: HOW MUCH LEFT 17295 BUM 0:06
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

110- 7 MISTAKES 10200 SON 2:18
This song uses a vaudeville setting to convey the message that everyone makes mistakes and everyone learns from mistakes.

GOAL 1: A C - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

110- 8 BUILDING GO BOOM 12130 LAF 2:07
A con-artist attempts to sell a 12 story building (with 10 ft. between stories) to a country bumpkin before the building's scheduled demolition.

GOAL 1: C - - GOAL 2: -0- GOAL 3: B1 C1 -0 PS: -
-0 -0

110- 9 (ICE CREAM) LOGO 16630 BUM 0:07
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

110-10 ICE CREAM STORE: CALORIES 10130 STU 3:10
A dieting woman enters an ice cream store run by a Valley Boy who uses a bar chart and percents to compare the calories of the various frozen treats.

GOAL 1: A C - GOAL 2: A1 B3 D1 C1d GOAL 3: A5 A3 D1 PS: X
F6 -0

110-11 (ME AND MY SHADOW INTRO) LOGO 17470 BUM 0:09
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

110-12 ME AND MY SHADOW 13660 STU 2:36
Debbie Allen discusses dimensionality by comparing her own 3-dimensionality with the 2-dimensionality of her shadow.

GOAL 1: C - - GOAL 2: -0- GOAL 3: G1 C2 -0 PS: -
-0 -0

110-13 MATHNET-CASE OF THE MISSING BASEBALL-5 10760 NET 6:41
The Mathnetters use a floor plan to successfully locate the missing baseball.

GOAL 1: - - - GOAL 2: A1 B1 C1a C3b GOAL 3: G4 G6 -0 PS: X
-0 -0

231- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

231- 2 PIECE OF THE PIE # 6 (SEASON 2) 20400 GAM 7:31
Two teams alternate guessing the most common answers to the survey question "Name something you identify by its smell." The team that accumulates the greater percentage wins the game.

GOAL 1: A C - GOAL 2: A1 B3 B6 C3b D2 GOAL 3: A5 B1 D1 PS: X
F6 -0

231- 3 DIRK NIBLICK: DO NOT FOLD, SPINDLE, PT.1 21110 ANI 6:07
Dirk confronts a salesman about the legitimacy of his sales tactics. He pressures his customers into buying three sixty-minute audio tapes for $3.95 instead of two ninety-minute tapes for $2.95.

GOAL 1: A - - GOAL 2: A1 B1 B3 C1c D1 GOAL 3: B5 -0 -0 PS: X
-0 -0

231- 4 AREA-VER.2 20940 ANI 0:20
This short animation shows that the area of a rectangular figure is the product of its length and width.

GOAL 1: - - - GOAL 2: -0- GOAL 3: C1 C2 -0 PS: -
-0 -0

231- 5 DIRK NIBLICK: DO NOT FOLD, SPINDLE, PT.2 21111 PAR 2:45
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

231- 6 MATHMAN: INEQUALITY T+40 < 75 20100 ANI 1:43
Mathman plays a video game in which he must eat all numbers satisfying the inequality T+40 < 75.

GOAL 1: C - - GOAL 2: -0- GOAL 3: A4 B1 D1 PS: -
D4 -0

231- 7 MATHNET-CASE OF THE WILLING PARROT-1 20030 NET 8:44
The Mathnetters receive a call from Walter Treppling about a haunted mansion. The Mathnetters go to investigate, only to find that the mansion is owned by Little Louie -- a parrot.

GOAL 1: A C - GOAL 2: A1 B3 GOAL 3: C2 D2 G4 PS: X
-0 -0

232- 1 SHOW OPEN 15950 BUM 0:46
* -0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

232- 2 COMBO JOMBO 21400 SON 3:41
The song demonstrates the use of combinatorics to find the number of combinations of bands of several sizes given the number of each type player available.

GOAL 1: A - - GOAL 2: A1 A3 B4 C1c D1 GOAL 3: B1 E1 -0 PS: X
-0 -0

232- 3 DATA HEADACHE I 14311 STU 1:10
A woman uses a bar chart to organize her monthly expenses and rid herself of a data headache.

GOAL 1: A - - GOAL 2: -0- GOAL 3: F6 -0 -0 PS: -
-0 -0

232- 4 SQUARE ONE SQUARES # 4 20560 GAM 5:27
Two students try to determine which cast member is giving the correct answer to the questions: Grid with Star, and Futuristic Money.

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

232- 6 SQUARE ONE SQUARES # 4 QUESTION 1 20561 SOS -0-
What number belongs in the square with the star in it, if you count along the edge of a 10x10 grid?

GOAL 1: - - - GOAL 2: A1 B4 C1c C3a GOAL 3: B1 C2 C4 PS: X
-0 -0

232- 5 SQUARE ONE SQUARES # 4 QUESTION 2 20562 SOS -0-
Which of the two figures contains more triangles?

GOAL 1: C - - GOAL 2: A1 B4 C1b C1e GOAL 3: G6 -0 -0 PS: X
C2c C4a -0 -0

232- 7 POLYHEDRONS - 1 (TETRAHEDRON) VERSION 2 21580 ANI 0:24
This animation illustrates how an arrangement of triangles fold up into a 3-dimensional tetrahedron.

GOAL 1: B - - GOAL 2: -0- GOAL 3: G6 G1 G2 PS: -
-0 -0

232- 8 BLACKSTONE: DIME, PENNY, NICKEL 15537 STU 3:13
Blackstone uses a fundamental property of even and odd numbers to correctly identify which hand holds the dime and which holds the penny. His follow-up trick depends on psychology--not mathematics.

GOAL 1: - - - GOAL 2: -0- GOAL 3: B3 B1 -0 PS: -
-0 -0

232- 9 INFINITY (INFINITE REGRESS) 16250 ANI 0:41
The camera zooms in on Beverly sitting in a room with a picture of Beverly sitting in a room with a picture of Beverly sitting in a room -- to illustrate the idea of infinite regress.

GOAL 1: B - - GOAL 2: -0- GOAL 3: D1 G2 -0 PS: -
-0 -0

232-10 MATHNET-CASE OF THE WILLING PARROT-2 20031 NET 12:06
The Mathnetters find their ghost, Norman Tedge, hiding out in the mansion. Then they uncover a puzzle with a particular pattern which might prove to have a bearing on the missing money.

GOAL 1: - - - GOAL 2: A1 B1 B3 B4 B5 GOAL 3: A5 D2 F1 PS: X
C1b C3a F4 G6

233- 1 SHOW OPEN 15950 BUM 0:46
* -0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

233- 2 GHOST OF A CHANCE 11950 SON 4:20

At a haunted house, a pizza delivery boy finds himself in several threatening situations -- each of which has a different probability of escape.

GOAL 1: A C - GOAL 2: A1 A2 B5 B6 GOAL 3: F1 F3 -0 PS: X
-0 -0

233- 3 CLOSE CALL # 6 (SEASON 2) 20790 GAM 5:45

Students compete against each other trying to get the closest estimate to: Colored Balloons, Bananas on a Table, Slices of Bread in the Sandwich, and % of Audience with Pom Poms.

GOAL 1: C - - GOAL 2: A1 B2 C2a GOAL 3: A5 C1 C2 PS: X
C3 -0

233- 4 MATHMAN: INEQUALITY 20 > A+5 20090 ANI 1:32

Mathman plays a video game in which he must eat all numbers satisfying the inequality 20 > A + 5.

GOAL 1: C - - GOAL 2: -0- GOAL 3: A4 B1 D1 PS: -
D4 -0

233- 5 POLYHEDRONS - 2 (HEXAHEDRON) (VERSION 2) 21590 ANI 0:28

This animation illustrates how an arrangement of squares folds up into a 3-dimensional hexahedron.

GOAL 1: B - - GOAL 2: -0- GOAL 3: G6 G1 G2 PS: -
-0 -0

233- 6 PHONER: FIBONACCI SEQUENCE 15960 STU 2:08

Arthur has a one-sided telephone conversation where he writes down the Fibonacci Sequence -- a series of numbers beginning with 1 whose next term is generated by adding the two terms previous.

GOAL 1: A C - GOAL 2: -0- GOAL 3: B3 D2 -0 PS: -
-0 -0

233- 7 MATHNET-CASE OF THE WILLING PARROT-3 20032 NET 12:13

The Mathnetters are called in to solve the mystery of the missing parrot. Using problem solving skills and the Fibonacci sequence they are able to find the missing bird.

GOAL 1: B C - GOAL 2: A1 A3 B1 B4 B6 GOAL 3: B3 G6 -0 PS: X
C1e C3a C4a C4b -0 -0

234- 1 SHOW OPEN 15950 BUM 0:46

-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

234- 2 DIRK NIBLICK: GO WEST YOUNG MATH. PT.1 20250 ANI 5:53
Dirk comes to the rescue of townspeople being swindled when purchasing land. Although they receive the proper amount of land, it is not in dimensions suitable for building.

GOAL 1: A C - GOAL 2: A1 A2 B1 B3 B4 GOAL 3: C2 D2 -0 PS: X
C1b C2a D1 D2 -0 -0

234- 3 TESSELLATION ANIMATION:TILE 10740 ANI 1:13
This animation shows both hexagons tessellating alone and hexagons forming a tessellated pattern with a star shape. The final image shows the same tessellated pattern on a real-life tile mosaic.

GOAL 1: A B - GOAL 2: -0- GOAL 3: G3 G6 -0 PS: -
-0 -0

234- 4 DIRK NIBLICK: GO WEST YOUNG MATH. PT.2 20251 PAR 3:10
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

234- 5 MATHMAN: EXTRA SHORT 15660 ANI 0:27
Before Mathman can begin his video game, Mr. Glitch eats him.

GOAL 1: C - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

234- 6 STICK SQUARES - 1 13951 STU 0:25
Alison Smith uses toothpicks to make a square that is divided into 9 smaller squares and asks the audience how many squares there are in all.

GOAL 1: C - - GOAL 2: A1 B4 D1 D2 C1e GOAL 3: G6 -0 -0 PS: X
C4a -0 -0

234- 7 SUGAR RAY SKETCH 14770 STU 4:32
Battling for the Doggy Weight Championship, Sugar Ray Leonard figures out the weight of a dog by picking up the dog, weighing their total, and then subtracting his weight.

GOAL 1: A C - GOAL 2: A1 A2 A3 B3 B4 GOAL 3: C2 B1 -0 PS: X
D1 D3 C1e C4a -0 -0

234- 8 STICK SQUARES - 2 13952 STU 0:42
Alison Smith demonstrates toothpick square tricks to the viewing audience.

GOAL 1: C - - GOAL 2: A1 B4 D1 D2 C1e GOAL 3: G6 -0 -0 PS: X
C4a -0 -0

234- 9 MATHNET-CASE OF THE WILLING PARROT-4 20033 NET 11:03
The Mathnetters investigate the latest crime, a birdnapping -- namely, Little Louie. Their checking of resources leads them to Norman Tedge. They arrest him, but he escapes before their very eyes.

GOAL 1: - - - GOAL 2: A1 B1 B3 B6 C3b GOAL 3: B3 -0 -0 PS: X
D3 -0 -0

235- 1 SHOW OPEN 15950 BUM 0:46
-0-

GOAL 1: - - - GOAL 2: -0- GOAL 3: -0 -0 -0 PS: -
-0 -0

235- 2 ONE BILLION IS BIG 20850 SON 3:14
The Fat Boys sing about one billion and its relative magnitude compared to one million.

GOAL 1: - - - GOAL 2: A1 B2 B4 C1b GOAL 3: A1 A2 -0 PS: X
C2a D1 -0 -0

235- 3 DIRKLET: TRIPLE PLAY PROMO/TRIANGLES 21190 ANI 0:47
Dirk says he loves Triple Play and wonders if all triangles are equilateral.

GOAL 1: - - - GOAL 2: A1 GOAL 3: G6 -0 -0 PS: X
-0 -0

235- 4 EB: PONG GAME 15180 ANI 0:19
This animation illustrates billiard geometry and shows a ball rebounding from wall to wall before finally exiting the one opening.

GOAL 1: B - - GOAL 2: -0- GOAL 3: G2 G6 -0 PS: -
-0 -0

235- 5 TRIPLE PLAY # 9 20720 GAM 6:15
Two students compete against each other trying to cover the vertices of an equilateral triangle. Multiplication and Addition sentences must be created in order to cover a vertex.

GOAL 1: C - - GOAL 2: A1 A3 B4 C1b GOAL 3: B1 G6 -0 PS: X
C2c C3a -0 -0

235- 6 MATHMAN: SQUARE NUMBERS #1 20040 ANI 1:39
Mathman plays a video game in which he must eat all square numbers.

GOAL 1: C - - GOAL 2: -0- GOAL 3: B2 -0 -0 PS: -
-0 -0

235- 7 SQUARE ONE PUZZLER: SALARY (.5 vs .25) 21150 ANI 0:48
A short animation puzzler: Which is more, .5 or .25?

GOAL 1: - - - GOAL 2: A1 B4 C1b C1e GOAL 3: A3 A4 D1 PS: X
C2a -0 -0

235- 8 SALE, THE 14060 LAF 1:07
Two girls figure out what twenty percent off a thirty dollar dress is.

GOAL 1: A C - GOAL 2: A1 A2 B4 C2c GOAL 3: A5 B1 A3 PS: X
-0 -0

235- 9 MATHNET-CASE OF THE WILLING PARROT-5 20034 NET 11:58
The Mathnetters use Walter Treppling's assistance in making a conversion between the Fibonacci sequence and a pattern of tiles. Solving this pattern leads them to the hidden fortune left in the will.

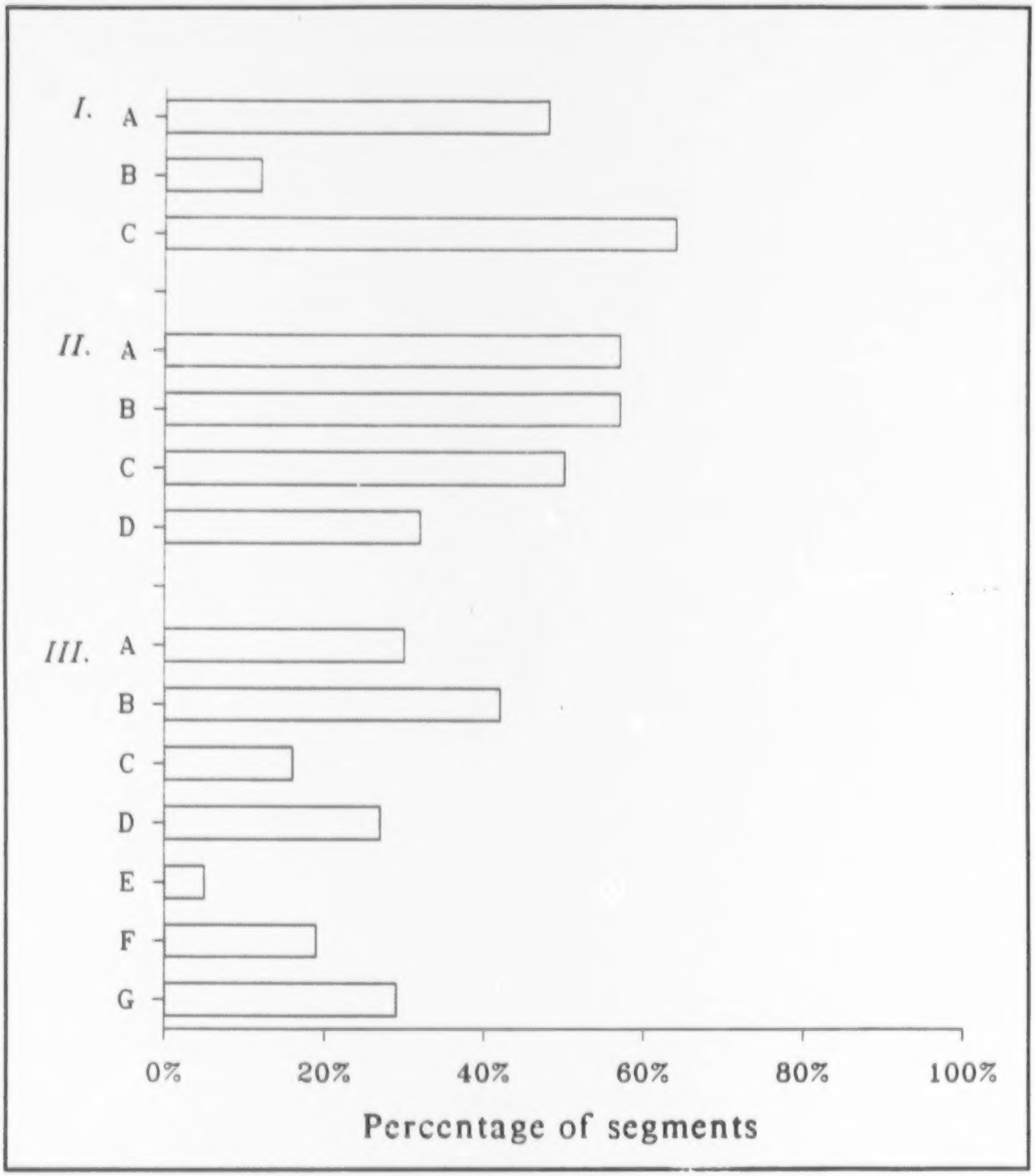
GOAL 1: B C - GOAL 2: B1 B3 C1b C1e C2a C2c C3a C3b GOAL 3: B1 B3 D2 PS: X
-0 -0

* Dirklet was deleted
** Show 209 was replaced with Show 239
*** "To Heck and Back" was replaced with "Itty Bitty Business" (Show 218)
**** "To Heck and Back" was replaced with "Mall or Nothing at Mall" (Show 217)
***** "Diet Lite Wet" was deleted

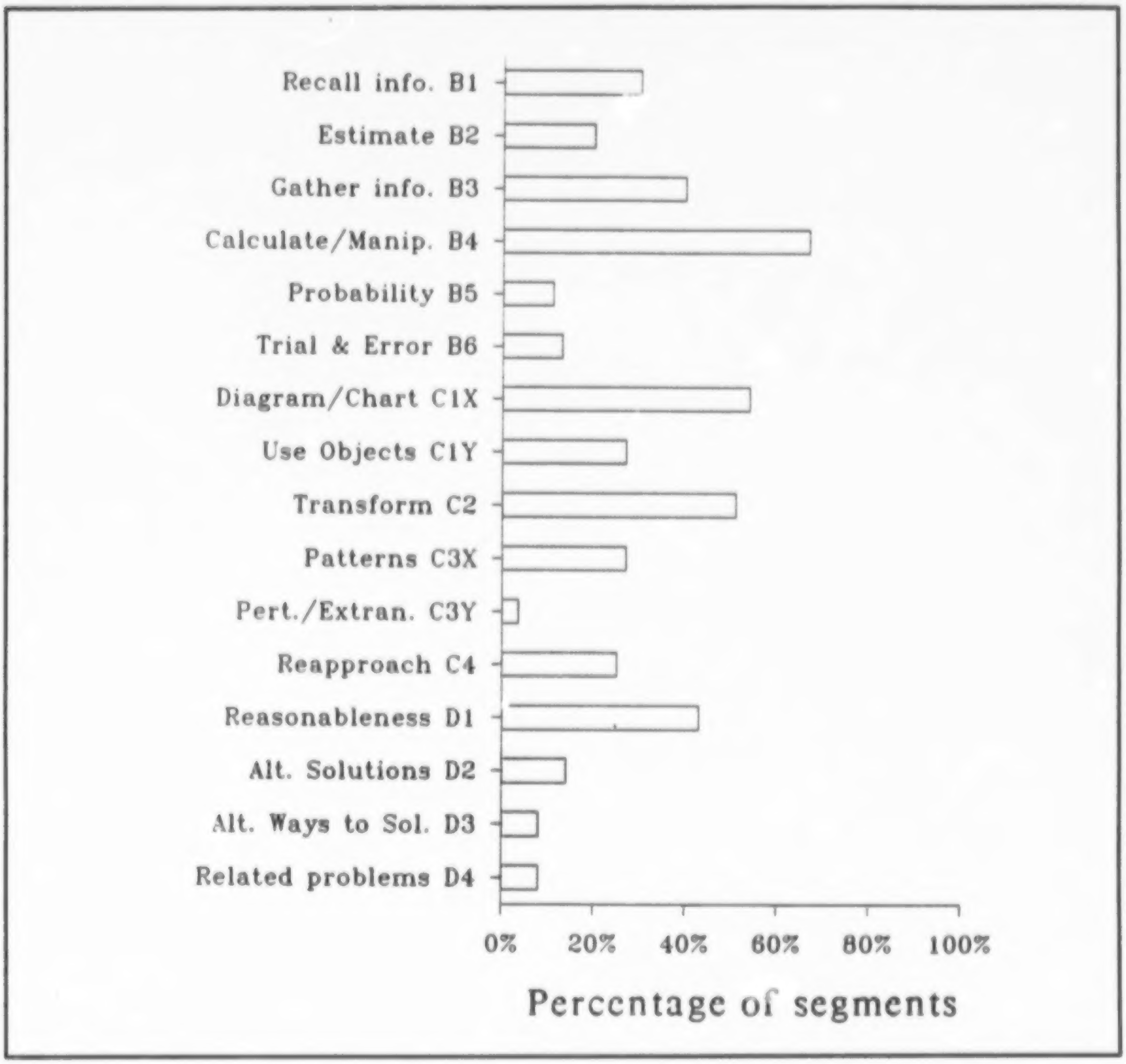
APPENDIX I.C:

Mathematical Content of Programs

The following graph represents the percentage of problem-solving segments, among those shown to viewers, that met the criteria of Goals I - III of **SQUARE ONE TV**:



The following graph focuses more closely on Goal II. It presents the percentage of problem-solving segments, among those shown to viewers, portraying Goal II actions and heuristics pertinent to the study (i.e., for which children's behavior was coded in the PSAs).



Note: Because the categories of actions presented in this table were also used to code children's behavior, they are slightly different than those described in the goals of the series (Appendix I.A). Most notably, children's behavior was not coded under Goal IA (the formulation and statement of a problem) since the problems they worked on had already been formulated for them by an interviewer. This is discussed more fully in Volume II, Chapter 4 (The Coding System).

APPENDIX I.D:

The Preptalk

PREPTALK

Introductions

Introductions of personnel by name.

Goals

We are a group of people -- we're called "researchers" -- who are interested in finding out how kids think about things. We do this by visiting lots of schools and talking with lots of kids. We've never worked in Texas before, and we're very happy to be here in Corpus Christi.

The reason why we want to know how kids think about things is so that we can help teach people better and improve the ways people learn. We think that the best way to find out about how kids think is to talk with them, so that's just what we're going to do this week. Basically, we're here to learn from you, and we appreciate your taking the time to meet with us.

As I said before, we want to know how each of you thinks about things. We do this by using what we call non-directive interviews. What do you think a non-directive interview is? [Allow some time for responses.]

A non-directive interview is an interview where we're not trying to lead you in any particular direction. We will ask you lots of questions so that we can get a clearer idea of what you have in mind. We often ask for more detail or explanation because it is hard to get inside of your head to know exactly what you are thinking. So, you really have to explain to us what is going on in your mind. When we ask questions, it does not mean that we disapprove or approve of what you're saying. We are not trying to lead you in any particular direction. There is no right or wrong here. We want to know what you think.

Role Playing

Let me give you an example of what I mean. Let's pretend that X is the interviewer and Y is one of you. [The parts of X and Y were played by two of the researchers.]

X: Y, can you tell me who your favorite person in history is?

Y: George Washington.

X: Uh-huh.

Y: He came over on a boat from Spain and discovered America.

X: Okay.

Y: And then he went on to become the first president of the United States. He did great things.

X: Can you tell me what you mean by 'great things?'

Y: Well, when he decided to make a flag...that was a great thing.

X: I see.

Okay, stop. Notice that when Y said "George Washington" X just said "uh-huh." X purposefully didn't give any hint about what s/he thinks about George Washington. S/he may think that Y's choice is a good one, or maybe not; but whatever s/he thinks, she's not telling Y about it; she's just encouraging Y to go on talking. Even when Y says that what she likes about George Washington is that he discovered America, X just says "Okay." She doesn't say anything like "I think you're confusing George Washington with Christopher Columbus." All that the interviewer is doing is trying to get you to talk about what you think.

You've all been chosen because you're different from one another. Since we're interested in the different ways that each of you thinks, it's really important that everyone gets a chance to think about the things that we will be doing and the questions that we will be asking on their own and in their own way, without knowing what someone else thinks about it. For that reason, we'd like it if you didn't talk to each other about what we have done together until the end of the week. Feel free to talk to other people outside of school about it, just not to the other people here or in school, okay?

Other Points

1. Working alone. Sometimes parts of our interview are so non-directive that we'll walk off and leave you alone for a while to think about something. When we do, you can take as much time as you like. We are not timing you.

2. Equipment. In all the interviewing that we do we bring along a kit of equipment. [Show kit] It's the same stuff for all of the interviewing that we do. Some of you might find that some of these things will be helpful to you, and maybe some of it won't. But it is available, and you can use it if you want to. The kit consists of:

a pad of white lined paper
pencils; colored pencils; felt tipped pens
calculator
ruler
gummed stickers (discs)
protractor
scissors

[Say what is in the kit. The envelope should be passed around and the kids encouraged to pick up the materials.]

3. Taping. We see so many kids that it's impossible to remember what everyone says. So we videotape all the interviews so that we can play them when we get back to our offices. No one else sees the tapes. No one at this school will see the tapes (teachers or other kids). It's all confidential.

Schedule

We will see each of you for two days in a row, for about fifty-five minutes each day. We will see A, B, C, D, E and F [referring to individual children] on Tuesday and Wednesday, and G, H, I, J, K and L [referring to individual children] on Thursday and Friday. We've prepared these cards to tell you when we are scheduled to meet with you. [Pass out cards.] Does anyone have any questions about his or her card? [Answer questions.] We won't all be here to work with you this week, you'll only be meeting with two of us. One of us on one day, and one of us on another day. Any questions? Okay, we'll see some of you tomorrow and Wednesday, and the rest of you this Thursday and Friday.

Children's Problem-Solving Behavior and Their Attitudes Toward Mathematics: A Study of the Effects of **SQUARE ONE TV**

VOLUME II

The Effects of **SQUARE ONE TV** on Children's Problem Solving

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PREFACE

This is the second of five volumes that describe an evaluation entitled "Children's Problem-Solving Behavior and Their Attitudes Toward Mathematics: A Study of the Effects of **SQUARE ONE TV**." The study was designed to assess the effects of **SQUARE ONE TV** on children's use of problem-solving actions and heuristics and on their attitudes toward mathematics. In addition, children were interviewed about their opinions of and reactions to **SQUARE ONE TV** itself.

The contents of the five volumes are as follows:

Volume I: Introduction: Purpose and General Design of the Study
Volume II: Effects of **SQUARE ONE TV** on Children's Problem Solving
Volume III: Children's Attitudes Toward Mathematics and the Effects of **SQUARE ONE TV**
Volume IV: The **SQUARE ONE TV** Interview: Children's Reactions to the Series
Volume V: Executive Summary

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CHAPTER 1

CONCEPTUALIZATION

Introduction

The first volume of this report has provided an overview of the entire study, including a description of its overall purpose and its design and methodology. The reader is referred to that volume for this basic information. The purpose of the present volume is to describe in detail the part of the study that relates to problem solving.

The plan of this first chapter is as follows: We will start by reviewing and extending the Volume I description of the general context of U.S. mathematics education that surrounds the study. Next, we provide a more detailed description of **SQUARE ONE TV**, and particularly Goal II, as a response to the current condition of mathematics education in this country. This is followed by a discussion of some of the relevant issues in research, including a framework into which our view of problem solving can be placed. We conclude by presenting some issues related to assessing change in problem-solving behavior.

A Context of Mathematics Education

Both the creation of **SQUARE ONE TV** and any evaluation of the series are, in part, products of forces that are acting within a context of present-day mathematics education. This general context has already been described in Volume I of this report; some further background with respect to problem solving is provided below.

Increased concern with problem solving. As we suggested in Volume I, most elementary school mathematics curricula place an extremely heavy emphasis on computational skills and low-level kinds of routine word problems. (There are exceptions, of course, notably including the Comprehensive School Mathematics Program (CSMP, 1989) and the Real Math text series (Willoughby, Bereiter, Hilton & Rubinstein, 1985).) This emphasis on computation has been viewed by many professionals in mathematics education as a serious deficiency, and hence there have arisen a number of proposals for reform.

Over the past several years, one of the groups that has played a significant part in the reform movement has been the National Council of Teachers of Mathematics (NCTM). NCTM is the world's largest professional organization concerned with the teaching of mathematics. Two of its publications merit particular mention here.

First, NCTM's Agenda for Action (1980) set forth a list of major concerns for the mathematics education community to address during the decade of the 1980s. Principal among these was an increased emphasis on problem solving: indeed, the Agenda asserts that "problem solving must be the focus of school mathematics" (NCTM, 1980, p. 2). Regrettably, NCTM's position has not been as widely adopted as one might wish, and often the rhetoric of problem solving has been misapplied to the same kinds of rote, computational exercises that characterize much of mathematics instruction in this country. Hence a need was seen for a more detailed document, spelling out more explicitly the kinds of mathematics that would be appropriate for the 1990s.

The more detailed document is the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989). Here a clarion call is made for a broader conception of the content of elementary mathematics and the inclusion of genuine problem solving at all grade levels, from kindergarten through high school. A clear distinction is made, too, between the stilted "word problems" of the typical textbook and the open-ended, nonroutine problems that require deeper thinking:

> The nonroutine problem situations envisioned in these standards are much broader in scope and substance than traditional word problems, which provide contexts for using particular formulas or algorithms but do not offer opportunities for true problem solving. Real-world problems are not ready-made exercises with easily processed procedures and numbers. Situations that allow students to experience problems with "messy" numbers or too much or not enough informa-

> tion or that have multiple solutions, each with different consequences, will better prepare them to solve problems they are likely to encounter in their daily lives. (NCTM, 1989, p. 76)

Recently, NCTM has been joined in its efforts to improve mathematics teaching and learning by a new organization, the Mathematical Sciences Education Board (MSEB). Unlike NCTM, MSEB is not a professional organization of teachers; rather, it is affiliated with the National Research Council (NRC), part of the National Academy of Sciences. Its membership includes a variety of professionals and laypeople who are concerned with mathematics and mathematics education, and its charge extends NCTM's to include mathematics through graduate school and beyond. Despite its wider constituency, MSEB has echoed and elaborated upon many of the same themes voiced by NCTM, as evidenced by two recent reports (National Research Council, 1989, 1990).

The first of these reports, Everybody Counts, is subtitled "A Report to the Nation on the Future of Mathematics Education." It describes the myriad factors that affect mathematical education from the earliest years to adulthood -- changing demographics, changes within mathematics itself, the growing role of technology, burgeoning international economic competition, and so on. The relatively brief section on elementary education supports the conclusion of the NCTM Standards by urging that "children (and teachers) [move] beyond narrow concern for school-certified algorithms for arithmetic [and] become mathematical problem-solvers" (NRC, 1989, pp. 46-47).

The second report is entitled Reshaping School Mathematics: A Philosophy and Framework for Curriculum (NRC, 1990). One of its principal aims is to propose a philosophy that views mathematics as a "science and language of patterns" (NRC, 1990, p. 12). Starting from this basic premise, the authors derive a number of conclusions about the content of mathematics and how it should be taught. One conclusion concerns the nature of problems that deserve to be incorporated into instruction:

> This perspective [of mathematics as a science and language of patterns] makes clear that students need to experience genuine problems -- those whose solutions

> have yet to be developed by the students (or even perhaps by their teachers). Problem situations should be complex enough to offer challenge, but not so complex as to be insoluble. (NRC, 1990, p. 14)

Taken together, then, these documents promulgate a vision of mathematics that, at the elementary-school level, is characterized by increased emphasis on solving nonroutine problems that involve mathematics beyond rote arithmetic.

Goals of **SQUARE ONE TV**

Recall from Volume I that problem solving is not the only area of concern for the reform movement: attitudes and mathematical content are of interest as well. The planning and production of **SQUARE ONE TV** started in the midst of widespread discussions in the mathematics education profession about the need for reform (but before the actual publication of the NCTM and MSEB documents cited above). As a result, the three principal goals of **SQUARE ONE TV** reflect these concerns directly; the goals are these:

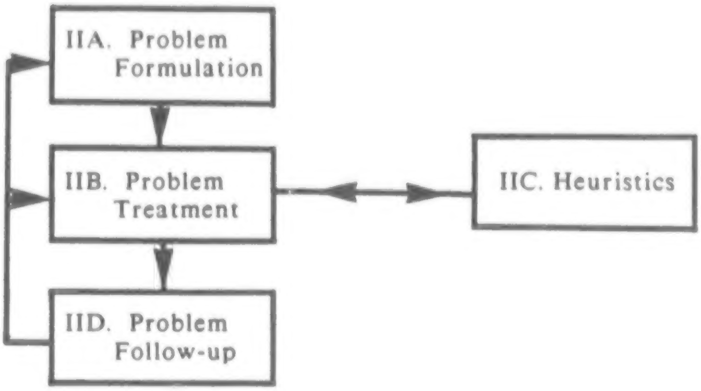
> Goal I: to promote positive attitudes toward, and enthusiasm for, mathematics;
>
> Goal II: to encourage the use and application of problem-solving processes; and
>
> Goal III: to present sound mathematical content in an interesting, accessible, and meaningful manner.

Goal II, on problem solving, is of primary interest in the present volume. For the purposes of the series, we think of problem solving as a potentially iterative process, as Figure 1.1, on the following page, suggests. Figure 1.1 is intended to be read as a kind of flowchart: Once a problem has been formulated, the problem-solver can treat the problem in a variety of ways. The interaction between Problem Treatment and Heuristics is two-way: in one direction, problem treatment might be assisted by the use of certain heuristics, and, reciprocally, progress in treating a problem can affect which heuristics are selected as appropriate. After Problem

Treatment there is a stage of Problem Follow-up wherein one's solution(s) can be reconsidered or extended. If needed, this can be followed by a return to Problem Formulation or to Problem Treatment.

Figure 1.1

Flowchart of Subgoals of Goal II



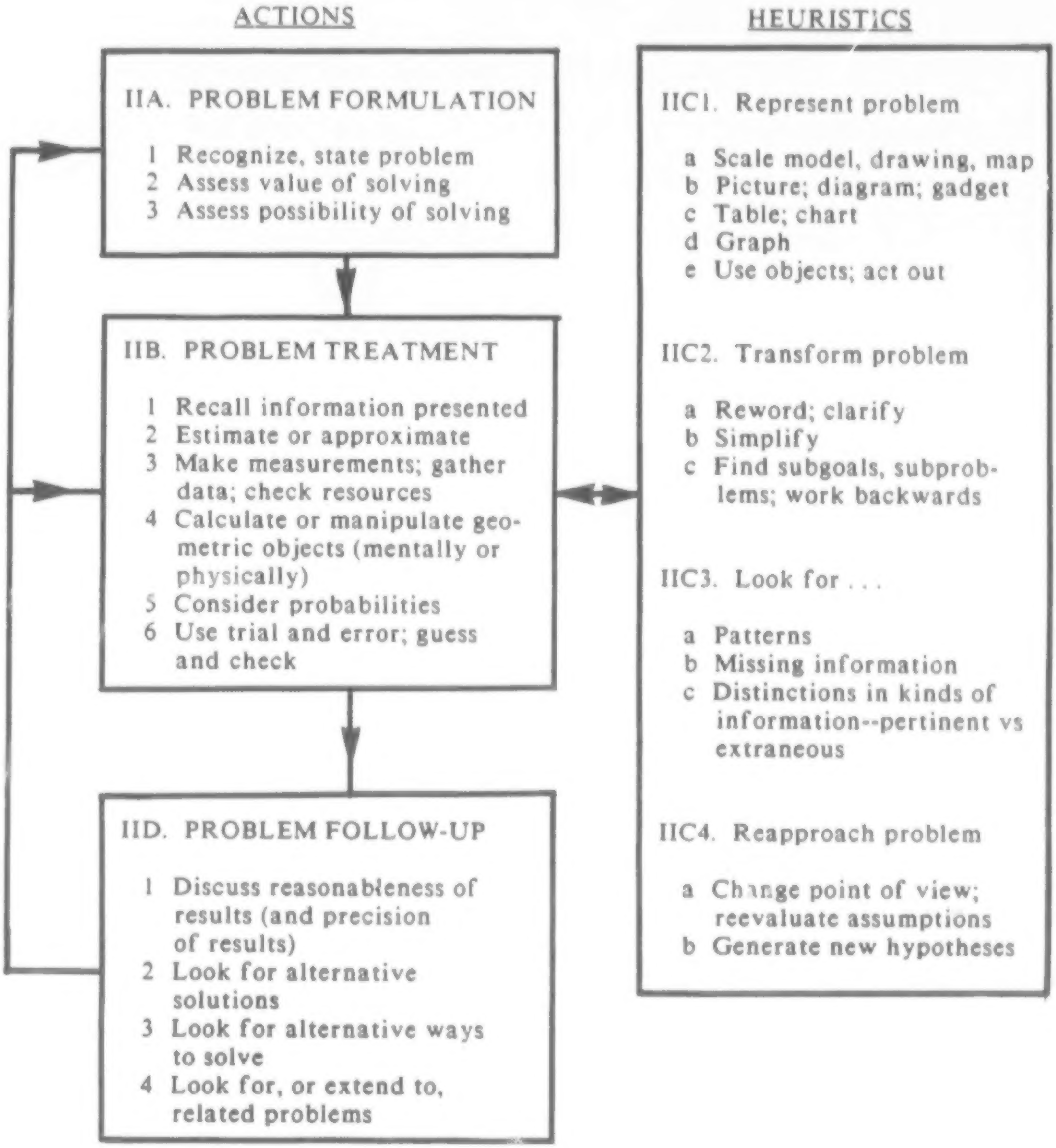
This general conception of the problem-solving process is rooted in the work of George Pólya (1957, 1962) and his followers (e.g., Fennell, 1983; Wickelgren, 1974). Over the past few years it has been adapted by **SQUARE ONE TV**'s Content Department staff to provide a framework for the development of the series.

The result of this adaptation has been a further breakdown of the model into a more detailed specification of what is meant by Formulation, Treatment, Heuristics and Follow-Up. The current version of the complete diagram appears in Figure 1.2.

Two points on nomenclature before proceeding: (a) Henceforth we will refer to Problem Formulation, Treatment, and Follow-Up (the left column of Figure 1.2) as problem-solving "actions," to distinguish them as a group from "heuristics." (b) Occasionally we will need to distinguish between levels of subgoals. In such cases we will refer to Problem Follow-Up (IID), for instance, as a subgoal; levels further down (e.g., Discuss reasonableness of results, IID1) are subsubgoals. When the level is clear from context we simply use "subgoal."

Figure 1.2

The Complete Goal II Flowchart


ACTIONS
HEURISTICS
IIA. PROBLEM FORMULATION
1 Recognize, state problem
2 Assess value of solving
3 Assess possibility of solving
IIB. PROBLEM TREATMENT
1 Recall information presented
2 Estimate or approximate
3 Make measurements; gather data; check resources
4 Calculate or manipulate geometric objects (mentally or physically)
5 Consider probabilities
6 Use trial and error; guess and check
IID. PROBLEM FOLLOW-UP
1 Discuss reasonableness of results (and precision of results)
2 Look for alternative solutions
3 Look for alternative ways to solve
4 Look for, or extend to, related problems
IIC1. Represent problem
a Scale model, drawing, map
b Picture; diagram; gadget
c Table; chart
d Graph
e Use objects; act out
IIC2. Transform problem
a Reword; clarify
b Simplify
c Find subgoals, subproblems; work backwards
IIC3. Look for . . .
a Patterns
b Missing information
c Distinctions in kinds of information--pertinent vs extraneous
IIC4. Reapproach problem
a Change point of view; reevaluate assumptions
b Generate new hypotheses

The subsubgoals within Problem Formulation, Treatment and Heuristics are largely self-explanatory. The only term that is not fairly standard is "gadget," which appears in subsubgoal IIC1b. A gadget is a physical embodiment of a concept or process that is used for illustration or clarification in **SQUARE ONE TV** sketches. In a sense, a gadget is a three-dimensional analogue of a diagram. Examples of gadgets include fanciful machines such as the "place value holder" designed to show the relationships among ones, tens, hundreds, and so forth, and the "function machine" used to show how numbers are transformed into other numbers.

Problem Follow-Up is an important feature of Polya's scheme (1957), but it is not often emphasized in elementary school instruction. Since it is less widely recognized than the other subgoals, it deserves some further explanation here.

As one can see from the detail provided in Figure 1.2, there are essentially two aspects of Problem Follow-up: reconsideration of the same problem (by seeing that one's results are unreasonable, for instance, or that one's answer is only one of several possibilities); or consideration of a different problem (perhaps as a variant or generalization of the original problem). Both aspects are important. The first can lead the problem solver to the realization that his or her initial answer is incorrect or incomplete, thus providing an opportunity to amend it. The second aspect of problem follow-up is important as a way in which students can come to recognize mathematics as a growing, creative enterprise. (Also, of course, the generalization of solutions to new problems and applications is itself an essential avenue through which mathematics develops as a discipline.)

The NCTM Standards recognizes both of these aspects (that is, follow-up leading to the same problem and follow-up leading to a different problem) when it calls for "varied experiences with problem solving as a method of inquiry and application so that students can ...verify and interpret results with respect to the original problem situation; [and] generalize solutions and strategies to new problem situations...." (NCTM, 1989, p. 75).

The full description of Goal II found in Figure 1.2 is used in essentially three ways in the development of **SQUARE ONE TV**: (a) to provide guidance to the production staff (writers, lyricists, etc.) as they incorporate aspects of problem solving into segments within a program; (b) to classify and analyze the content of segments once they have been produced;[1] and (c) to guide research and evaluation connected with the series.

Research in Mathematical Problem Solving

Concomitant with increased concern about U.S. mathematics education over the past 15 years has been increased interest in research in mathematics education. The research has become a vast body of literature, involving learners ranging from preschoolers through adults. (As an indication of the volume of this research, the most recent annual summary of the published research in the field (Suydam, 1990) lists a total of 446 articles and doctoral dissertations that appeared in 1989.) We will highlight some aspects of the research that are of particular relevance to the current study.

Historically, research in mathematics education has often stemmed from practical concerns related to the teaching of particular subject matter central to existing curricula. Good examples of this include Carpenter and Moser's investigations of how first graders deal with simple word problems in addition and subtraction (Carpenter & Moser, 1982); the work of Behr, Post and their colleagues on problem solving in the domain of rational numbers in the middle grades (Behr, Wachsmuth, & Post, 1985; Post, Behr, & Lesh, 1986; Post, Wachsmuth, Lesh, & Behr, 1985); research in algebra word problems (e.g., Wagner & Kieran, 1989); the construction of proofs in high school geometry (e.g., Usiskin, 1982; Senk, 1985); and calculus (e.g., Ferrini-Mundy, 1987).

In a sense, the present research on the effects of **SQUARE ONE TV** is consonant with this tradition of concern with curriculum. That is, the mathematical community has perceived a need to broaden the presentation of mathematics to children in the upper elementary grades; the research described here might be used for the practical purpose of moving toward the realization of that goal. Indeed, some research findings have had the effect of forcing a reconsideration of widely accepted underlying assumptions about the subject matter, how children learn it, and how it might be taught.[2] We will return to these themes in Chapter 6, after our own findings have been presented.

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[1] All **SQUARE ONE TV** segments from the first three seasons have been analyzed with respect to their Goal I and Goal III content as well as their Goal II content; the results of all of these analyses can be found in Schneider, Miller, McNeal & Esty (1990).

Problem-solving research vis-à-vis our view of problem solving. A substantial portion of the research in mathematics education has addressed mathematical problem solving (see Charles & Silver, 1988, Schoenfeld, 1985, or Silver, 1985, for recent reviews of this literature). To better understand problem solving on **SQUARE ONE TV** and how it is being investigated in this study, we will consider some relevant examples of the work of other researchers.

Many researchers have adopted a fairly broad view of the components that must be taken into account in describing mathematical problem solving. For example, Schoenfeld (1983) describes three categories of knowledge and behavior needed to characterize human problem solving:

> Category I: Resources (knowledge possessed by the individual, that can be brought to bear on the problem at hand). E.g., facts and algorithms; relevant competencies, including the use of routine procedures, "local" decision-making, and implementing "local" heuristics.
>
> Category II: Control (selection and implementation of tactical resources). E.g., monitoring; assessing; decision-making; conscious metacognitive acts.
>
> Category III: Belief Systems (not necessarily conscious determinants of an

---

[2] Carpenter and Moser's (1982) research is a good example here. They found that kindergarten children can solve addition and subtraction word problems by modeling them with objects (chips, for example) before they have had any formal instruction (including drill and practice) in the so-called "number facts" like $2 + 3 = 5$. Many educators of this age group had based their instructional programs on children's first mastering these facts and only then applying them to the typical first grade "word problems". Implications for teacher education are significant (Peterson, Fennema, and Carpenter, 1988-89).

> individual's behavior). E.g., about self; about the environment; about the topic; about mathematics. (p. 15)

Other researchers have posited similar breakdowns of factors involved in problem solving. For example, Lester (1982) asserts that:

> Successful problem solving is a function of at least five components: 1. mathematical knowledge and experience, 2. skill in the use of a variety of generic "tool" skills (e.g., sorting relevant from irrelevant information, drawing diagrams, etc.), 3. the ability to use a variety of heuristics known to be useful in mathematical problem solving, 4. knowledge about one's own cognitions before, during and after a problem-solving episode, and 5. the ability to maintain executive control (i.e., to monitor and regulate) of the procedures being employed during problem solving (p. 59).

In a later paper (Lester, Garofalo, & Kroll, 1989), these are supplemented by four kinds of "noncognitive and metacognitive factors...: affects and attitudes, beliefs, control, and contextual factors" (p. 75). (One of these, the metacognitive factor of "control," had been discussed in the 1982 paper.) While there are differences in the details of Schoenfeld's and Lester's conceptualizations of factors underlying problem solving, nonetheless the overlap is substantial. For our purposes here, we take Schoenfeld's (1983) model as a backdrop against which to present our own view.

All three of Schoenfeld's categories appear in some sense in the production of **SQUARE ONE TV**. Let us begin with Category III. The "Belief Systems" of Category III -- about oneself and about mathematics -- are embedded in Goal I of **SQUARE ONE TV**, which is "to promote positive attitudes and enthusiasm for mathematics." We will explore these issues later, in Volume III.

Category II, "Control", deals with issues related to choices made in the problem-solving process and the metacognitive assessment of one's progress through a given problem. The modeling of this type of thinking is not an explicit goal of **SQUARE ONE TV**. However, certain aspects of it are captured in Goal II, for example, in reapproaching a problem or following up on a solution. When this type of behavior is demonstrated in the series, it is often in the form of a comment from one character to another about the use of some particular heuristic technique.

Related to the monitoring and decision-making of Category II is the issue of the sequence in which individual problem-solving behaviors are selected and used. Although the modeling of sequence per se is not a goal of **SQUARE ONE TV**, sequence does appear in the series. **SQUARE ONE TV** scripts are based on a logical progression through a problem, typically starting from problem formulation and going through some kind of problem treatment (perhaps augmented by some heuristics) with, in some cases, some follow-up to a solution; when necessary, one or more parts of this process are repeated. These sequences are in part the natural result of the way in which their components are defined; for instance, one cannot have "problem follow-up" until there is a problem to follow up.

Some researchers have studied in microscopic detail the sequences in which subjects use individual problem-solving behaviors (e.g., Lucas, et al., 1984). This approach was not appropriate for our study, however, for two reasons. First, as we have already suggested, sequences are not a major focus of **SQUARE ONE TV**. Second, given the structure of Goal II and the coding system that evolved from it, it was not feasible to determine unambiguously the students' sequences of behaviors.[3] Thus we have chosen in the present study to examine the individual problem-solving behaviors themselves, as opposed to the sequence in which they occur.

This brings us to Schoenfeld's Category I, "Resources." The resources of this category correspond to individual actions and heuristics within the boxes of the model presented in Figure 1.2 (for example, considering probabilities, estimating, and making a diagram or chart). Because Goal II of **SQUARE ONE TV** is to "encourage the use and application of problem solving processes," this category is the one most relevant to the series' view of problem solving and to the present research study.

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[3] See Chapter 4, on the coding systems, for more detail on this.

Assessment of Mathematical Problem Solving

Our interest in children's problem-solving behavior in the present study goes beyond merely describing it: we are concerned with assessing the effects of a particular intervention -- viewing 30 programs of **SQUARE ONE TV**. In addition, we wanted to be able to distinguish the effects of viewing **SQUARE ONE TV**, if any, from effects of other events that take place in children's lives -- for example, effects of classroom instruction in mathematics, or of simple maturation.[4] As a result, two sets of issues had to be addressed at the outset: (a) how best to assess changes in children's problem solving and (b) how to conduct a comparative assessment that would not be biased toward or against the intervention. We consider each of these points in turn.

A range of problem-solving assessments. One can identify two ends of a scale of assessments. On one end there are the multiple-choice, paper-and-pencil assessments that are often used in nationally normed achievement tests. At the other end are so-called "dynamic assessments" (Campione & Brown, 1987) in which an interviewer, working with an individual student, attempts to ascertain his or her learning potential. The technique involves providing the student with just enough probe questions and cues to enable him or her to reach a predetermined level of performance. The interviewer must tailor the cues and probes to the individual student, based on that student's responses as the interview proceeds; hence one interview can be very different from another.

Between these extremes lie a variety of other assessment techniques, including portfolio assessments (Wiggins, 1989), hands-on performance assessments (Baxter, Shavelson, Goldman & Pine, 1990), focused observations (Stenmark, 1989), and task-based interviews (Davis, 1984). Also, of course, hybrid assessments can be created -- for instance, task-based interviews supplemented by norm-referenced paper-and-pencil instruments.

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[4] An earlier television series about mathematics, called Infinity Factory, (Trevino & Muir, 1976) was aimed at roughly the same target audience as **SQUARE ONE TV**'s. The formal evaluation of that program (Harvey, Quiroga, Crane, & Bottoms, 1976) did not incorporate any control group, making it difficult to ascribe any changes from pretest to posttest unambiguously.

Assessments that are situated at various places along the continuum described above are appropriate for various purposes. For example, the multiple-choice, written instruments, perhaps augmented with a limited number of free response items, may be the apt choice if one's aim is to make comparisons across groups (e.g., classrooms, schools, states) on skills (such as routine computation) that lend themselves to that format. Certainly when large numbers of subjects are involved, the more easily and efficiently administered the test is, the more attractive it becomes from the standpoint of cost.

In contrast, dynamic assessment may be the most appropriate choice if one's aim is to gather instructionally relevant information about individual children, particularly children for whom other testing formats are problematic. Of course, to the extent that assessments require interviews to be conducted individually, they are likely to be relatively expensive and time-consuming to plan, administer, and analyze.

When we were considering this assessment initially, we realized that the effects of **SQUARE ONE TV** on children's inclinations to use problem-solving techniques might be subtle and hard to detect, even when the series was viewed every day over a six-week period. The series' producers had consistently attempted to avoid didactic presentations reminiscent of typical mathematics classroom instruction, and even the problem solving that occurs in the series appears mostly via example, not overt exhortation. Further, we were interested not only in children's abilities to find answers to problems, but also in the behaviors they used and the reasoning behind their choices. Thus it was apparent that simple paper-and-pencil measures of mathematical performance could not capture the sort of detailed information that we needed. On the other hand, we were not interested in designing instructional programs for individual children, and so the technique of dynamic assessment was equally inappropriate.

Instead, what we wanted was to create a detailed portrait of children's problem-solving performance, using the same kinds of mathematically rich, nonroutine problems that appear on **SQUARE ONE TV**. We wanted the resulting description of children's behavior to be fine-grained enough to reflect what might be subtle effects of the program, yet sufficiently replicable (from one child to the next) to allow comparisons between groups of children.

These considerations ultimately led us to adopt the technique of the task-based interview (Davis, 1984). In this technique the interviewer presents some problem or situation to a child and then, in an interactive discussion, explores in depth the child's reasoning. Since the pioneering work of Jean Piaget (e.g., 1952, 1956, 1960), the method has been used extensively in mathematics education research, and has proved to have enormous power to illuminate children's thinking processes. Our own previous experience with the technique (Peel, et al., 1987) had shown its ability to capture interesting and meaningful behavior.

Conducting comparative assessments. Consider the problem of assessing the impact of any sort of innovative program. Regardless of where one prefers to be on the range of assessments described in the previous paragraphs, one must be concerned with being fair both to the group that used the program and to the group that did not. Clearly the assessment instrument cannot be constructed so as to require exposure to the innovative program for successful performance. A good example of this was the work of Dougherty (1982) in connection with evaluations of the innovative Comprehensive School Mathematics Program. There, special tests were developed "to assess what are thought to be some of the underlying thinking skills of CSMP.... The tests do not contain any of the special vocabulary or techniques of the CSMP program and most of them are built around mathematical situations that are [equally] unfamiliar to both CSMP and Non-CSMP students" (p. 5). In our case, this meant that we had to ensure that none of the problem-solving instruments could involve or depend upon prior knowledge of **SQUARE ONE TV** -- its stories, characters, formats, or any special notations, methods or vocabulary. We will return to this issue in Chapter 3, on the construction of the Problem-Solving Activities.

CHAPTER 2

THE PILOT-TEST PHASE

Introduction

Over a period of eight months, from June, 1988 to January, 1989, intensive effort was devoted to the development of assessment instruments and techniques. Some of the issues that were addressed during this time were purely procedural, and applied generally to the entire study; these were described in Volume I of this report. Others were specific to the problem-solving part of the study, and these are described in the paragraphs that follow. Pilot-testing issues connected with the Attitude Interview appear in Volume III.

The conclusions that were reached as a result of the pilot testing were, of course, reflected in the final design of the instruments and procedures. These are explicated in Chapter 3 of this volume, which is devoted to the Problem-Solving Activities; the reader who is more interested in the study as it ultimately evolved than in its evolution per se can skip to the next chapter.

Two Interrelated Components

The pilot testing had two interrelated components. One was the incremental refinement of the content of the tasks and the wording of instructions, questions, etc.; this process took place throughout the eight months. The second component was a somewhat more formal attempt at a full pilot test, involving two schools with differing exposures to **SQUARE ONE TV**. These two components overlapped, but we treat them separately in the discussion below.

The First Component of Pilot Testing: Task Refinement

Three Problem-Solving Activities (PSAs), each having two versions, were used. All of the PSAs are nonroutine, open-ended problems, but they differ in their levels of complexity. The least complex problems, PSA A and PSA A', are combinatorial problems involving circus performers or stripes on a shirt. PSAs B and B' (problems of medium complexity) involve sorting party guests or price tags into piles that meet several conditions. Finally, PSAs C and C' (the most complex problems) ask children to determine what is wrong with a complicated mathematical game and to fix it. (More detail on the PSAs appears in the next chapter.)

The issues that were addressed during the process of task refinement were of two kinds: general issues related to all the PSAs and issues related to some specific PSA only. Issues that pertained to all three PSAs will be discussed here; those that were more closely tied to a specific PSA are addressed in Appendix II.A.

General Issues

Our overall aim was to create problems that would be engaging to children and that could form the basis of discussions between child and interviewer. During the course of the pilot-test phase we examined a number of general issues related to the creation and administration of these problems. These issues are summarized below, together with the conclusions we reached over the course of the pilot phase.

Size of the interview unit. The question here was: Should subjects work alone or in small groups (pairs or even triads)? Earlier research on **SQUARE ONE TV** (Peel, et al., 1987) had been done effectively with triads. During the pilot-test phase we experimented with both pairs and individuals.

Small groups have the great advantage of promoting interactions, particularly conversation about the task at hand; such conversation is valuable in understanding children's thinking.

On the other hand, there are a number of disadvantages involved in interviewing more than one subject at a time. One is that there is no way to know what interpersonal relationships might exist between members of a pair, and certainly no way to know how those might change from pretest to posttest. Conceivably one or both children in a pair might be more or less inclined to talk about the PSAs simply because of the presence of the other child.

Another major disadvantage, for this study, was that the Attitude Interview, part of which dealt with the child's reactions to the PSAs, was to be conducted individually; it seemed not to be feasible to separate a subject's reaction to the PSAs from his or her more general reaction to working with another individual.

Finally, it is difficult, if not impossible, to isolate each child's individual contribution to a group problem-solving effort. If the two partners are inclined to use the same strategy, then one child's use of that strategy obviates the need for the second child to express it. Alternatively, if the partners are inclined to pursue divergent strategies, then the use of one may preclude the use, or even the expression, of the other.

Some of the pilot testing was conducted with pairs of children, but after careful consideration of our experiences with pairs and individuals, and in view of our interest in examining children's use of _specific_ problem-solving actions and heuristics, we finally decided to conduct the interviews with individuals. The advantages of interviewing pairs of children were far outweighed by the disadvantages listed above.

Interaction between child and interviewer. Our overall goal in the PSA interviewing was to understand the children's thinking regarding the PSAs as fully as possible. We wanted the subjects to feel free to tell us everything they were thinking without encouraging their pursuit of any particular approach or giving any clues about what the interviewers thought were correct or desirable answers. Thus, we were striving to be supportive but nondirective.[5]

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[5] A comprehensive treatment of issues related to obtaining veridical reports of children's behavior is provided by Ericsson and Simon's (1984) _Protocol Analysis: Verbal Reports as Data_.

This goal was explicitly discussed right from the beginning, during the "Preptalk." As we have already described in Volume I, we met with all the subjects as a group the day before the individual interviews began and described our interviewing technique. The Preptalk included a mildly humorous mock interview, with one of the researchers playing the part of a subject. This was designed to demonstrate that the interviewer would have neutral reactions to a subject's wrong answers or misconceptions. The content and tone of the preptalk were developed over the pilot-testing phase. (The text of the Preptalk appears in Appendix I.D of Volume I.)

This theme was reemphasized in the pretest and posttest interviews. The goal of encouraging subjects to feel free to describe their thinking was pursued from the beginning, with the interviewer's introductory remarks. After trying several variations, the wording that we chose was:

> Before we start, I'd like to say that what we are interested in is how you think about the things that we are going to do today. At the end of each thing that I ask you to do, I will ask you lots of questions about what you did and what you were thinking. So, if you could try to remember what you are thinking as you do these different things, that would be really helpful. And when I ask you these questions at the end, I really do want to know everything that you thought -- your good ideas, your bad ideas and ideas that you think don't matter or aren't important. Is that clear? Any questions? Okay, let's begin.

Use of this introduction proved helpful in eliciting children's thought processes, but it was not completely sufficient. We found that subjects could not be relied upon to verbalize their thinking fully and consistently -- that is, to think aloud -- without repeated probing from the interviewer. However, we found that the constant presence of the interviewer was distracting at best: In some cases it seemed to convey the incorrect impression to children that quick answers were more desirable than thoughtful ones. In other cases, we found that the children were overly reliant on direction and feedback from the interviewers, thus inhibiting the expressions of their own problem-solving behavior.

After considerable experimentation, we converged on a hybrid strategy, used in all the

PSAs, that is essentially a variant of the classical task-based interview. The interviewer first posed the problem and made sure the subject had no questions, then left the interview table to sit somewhere else in the room, and finally returned (either at the subject's request or when a predetermined amount of time had elapsed) to discuss what the subject had done.[6]

As we proceeded with the development of interview formats, detailed interview protocols were constructed, along with a set of standard probe questions to be used to elicit further explanations and comments from the children. (Appendix II.B contains the protocols for the PSAs. Appendix II.C contains a list of the standard probes along with other interviewing guidelines.) Our interest in this study was in what children could do on their own, as opposed to how far they could get with guidance or structure from the interviewer. Further, we wanted results to be uniform (across different interviewers and from pretest to posttest), replicable, and independent of individual interviewers' orientations or mathematical backgrounds; hence the protocols were intended to be followed closely. Although the list of probe questions was fixed, the probes could be used at the interviewers' discretion to gain deeper insight into the children's thinking. While the standardization of the interview wording and the probe questions placed some limits on the interviewers' flexibility[7], nonetheless they had ample opportunity to explore the children's answers in depth.

In summary, then, all the components of our interactions with the children -- the Preptalk, the interview protocol, and the follow-up probes -- were designed to encourage them to describe their ideas as fully as possible, uninfluenced by what they might have thought we wanted to hear.

---

[6] Ericsson and Simon (1984) discuss in detail the differences between "think-aloud" techniques and ones (like ours) that are based on retrospection. They conclude that concurrent reports have the advantage of revealing the sequence in which the subject heeds (processes) information, but that "retrospective reports of specific cognitive processes [also] provide powerful means for gaining information about such processes" (p. 30). Further, Ericsson and Simon stress the importance of (a) obtaining the retrospective report soon after the problem solving, so that the cognitive processes used are still accessible in short-term memory, and (b) probing with questions that call for recollection of specific problem-solving steps (as opposed to summaries of general processes). Each of these points is incorporated into the present study.

[7] Certainly some researchers (e.g., Erlwanger, 1975; Krutetskii, 1976; Piaget, 1952) have had considerably more freedom than the interviewers in this study to redirect the course of an interview while it was under way.

Difficulty level. An important question was whether the PSAs were formulated at appropriate levels of difficulty. That is, can most children do something with them, or at least understand what the problem is about? On the other hand, are the PSAs challenging enough so that most subjects do not solve all of them completely? We experimented with successive refinements of the PSAs made during the pilot-testing period.[8] As described below, we eventually arrived at instruments for which the answers to these questions were affirmative.

The kit of materials. Certain approaches to each of the PSAs can be aided by the use of physical tools. We wanted to supply those tools without suggesting that their use would be necessarily helpful, expected, or desirable. Our solution here was to provide each interviewer with a standard equipment kit. The kit consisted of:

three pencils;
three pens of different colors;
one protractor;
one ruler;
one small four-function calculator;
a pad of white lined paper;
and several dozen gummed 1-inch paper discs of various colors.

This kit was described and passed around among the subjects during the Preptalk. Later, as part of each PSA, the interviewer mentioned the kit, reminding the child that he or she might or might not find its contents useful.

Context or not? Should the PSAs be presented with some sort of story line, or should they be free of any context? Certainly the use of a story context is consonant with the format of **SQUARE ONE TV** (in which mathematics is often embedded in various contexts), and there appeared to be other advantages to using a context as well. One was that surrounding the problems with a story context seemed more involving and engaging to the children in the pilot test. Further, a story gives the interviewer additional hooks on which to hang post-performance discussion. Finally, even though the stories are somewhat contrived, they provide some examples of how mathematics might be useful; and since usefulness was to be one topic for the subsequent attitude interviews, it seemed reasonable to provide some examples of usefulness that might be discussed later.

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[8] In some cases this involved minor adjustments of numbers within the problems (e.g., the number of stripes involved in PSA A); in other cases (particularly PSA C) several candidates for the problem were considered and subsequently rejected as too difficult. Details appear in Appendix II.A.

One disadvantage of tying the PSAs to story contexts was that the contexts might distract some children from the mathematical essence of the problem; that is, their solutions could be inappropriately dependent on, or limited by, the contexts. Another potential pitfall (one that we took pains to avoid) was contexts that might be more appealing or familiar to one group than another (e.g., girls vs. boys).

We experimented with various presentations of context, and concluded that the advantages described above outweighed the disadvantages. Hence each PSA was embedded in a very simple story situation. As it turned out, only a few of the subjects in the pilot study became so involved in the context of a particular PSA that they did not pursue the mathematics.[9]

Constructing variants of PSAs. How should the variants (e.g., PSA C and PSA C') be constructed to be mathematically and psychologically similar, yet different enough to be perceived as such by the subjects? Considerable effort was devoted to this issue. Our general approach here was to make each member of a PSA pair look superficially different from the other member, while keeping most, if not all, of the mathematical structure the same. In all cases this meant making the physical materials appear different. Details are provided in the next chapter and in Appendix II.D, where the exact nature of the differences are explained.

Ordering and timing of the PSAs. The issues here were: In what order should the PSAs be presented? How much time do they take? How many can be done at one sitting? After considerable experimentation we determined the following order, which we used with all subjects in both the pretest and the posttest.

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[9] Similarly, only a few of the children in the main study were distracted by the context.

> Day 1: PSA C (or C') followed by PSA B (or B'). (Approximately 55 minutes.)
>
> Day 2: A five-minute piece of the Attitude Interview in which the child discusses his or her experiences with the PSAs from the day before; followed by PSA A (or A'); followed by the rest of the Attitude Interview. (Approximately 55 minutes.)

A series of steps led to this final order: First, we knew that at least two interviewing sessions would be required (because of the amount of material involved), and we wanted to have at least one PSA in each session. Second, the parts of the Attitude Interview that concerned particular PSAs had to occur after the child had tackled those PSAs, because we wanted to be able to discuss the shared experience with the PSAs as examples of problem solving. Third, we knew that PSA C (or C') should occur on the first day so that the children would have a chance to think about it overnight. In fact, we thought that PSA C (or C') should be the first PSA on the first day, because pilot-test subjects seemed to find it the most engaging of the PSAs. We found that PSA A (or A') fit well with the Attitude Interview, in terms both of timing and of allowing a smooth transition to the main part of the Attitude Interview. Last, we determined that PSA B (or B') could fit into a 55-minute period with PSA C (or C').

In keeping with our general aim of encouraging children to think carefully about the problems, we told them that there was no need to rush, and to take as much time as they liked. In presenting PSA A (or A') we also said that we "may have to interrupt you before you finish, but that's okay." Pilot-test subjects were not troubled by these instructions, and were not upset by the interruption if, in fact, it occurred.

The amounts of time that were allotted for each PSA, including discussion, were approximately as follows: 40 minutes for PSA C (or C'); 15 minutes for PSA B (or B'); and 15 minutes for PSA A (or A'). The amounts of time that children actually spent working on the problems are presented in Chapter 5, on results.

Subjects' reactions to repeat interviews. There are two issues that we explored here: (a) How do subjects react to a second day of interviewing, with a different problem? (b) How do subjects react to a second round of interviewing at the posttest, six weeks or so after the first, with problems that are more or less the same as the earlier ones? Our work with the pilot-test children suggested no cause for concern here: most of them seemed about as enthusiastic and willing to cooperate in subsequent interviews as they were in the first.

The Second Component of Pilot Testing: The Informal Test

During the latter part of the eight-month pilot phase we conducted an informal test that involved children from two schools on New York City's Lower East Side. The schools were roughly matched on SES, ethnicity, and curriculum. In one school the children were encouraged to view **SQUARE ONE TV**, while children in the other school were not. Pretests and posttests were conducted with a sample of students (a total of 13) from both schools and analyzed for evidence of consistency and growth across time.

Overall, we found that more of the children in the "encouraged to view" school than in the other school showed increased use of appropriate problem-solving actions and heuristics from the pretest to the posttest. We emphasize, however, that the measures that we used were not as sophisticated as the scoring system used in the main study (see Chapter 4 of this volume). Also, in some cases the tasks and interview wording were still being refined, so that the tasks were not exactly the same across all the subjects. Nonetheless, the results of this pilot run were encouraging.

CHAPTER 3

DETAILED DESCRIPTION OF THE PROBLEM-SOLVING INSTRUMENTATION

Introduction

The purpose of this chapter is to describe in depth the Problem-Solving Activities (PSAs) that were used in the study, to relate them to each other, and to relate them to the goals of **SQUARE ONE TV**. Further, we will describe how the PSAs were administered as testing instruments.

Background

In Chapter 1 we discussed briefly some of the recent research related to mathematical problem solving. Researchers in this general area are understandably interested in being able to compare results across studies, and this has led to the development of systems by which problems or tasks used in research can be classified. One such general system is set forth in Task Variables in Mathematical Problem Solving (Goldin & McClintock, 1984). This scheme posits four sets of variables, all of which are relevant to the construction of the PSAs: (a) syntax variables (the relationships among the words or symbols involved in the problem); (b) variables of mathematical content (probability or geometry, for example) and non-mathematical context (the situation into which a problem is embedded and the mode (e.g., verbal, pictorial) in which it is presented); (c) variables that describe the structure of the problem or task (the mathematical relationships among the elements of the problem); and (d) variables that describe the heuristic processes (described, in our case, by the subgoals of Goal II) that the task evokes (Kulm, 1984). The relations among these variables and the criteria that we used in developing the PSAs will be explicated following a more detailed description of the PSAs.

Overview of the PSAs

The PSAs were designed to be a measure of children's problem-solving behavior by providing them the opportunity to grapple with some nonroutine problems in an open-ended context in which interviewers would provide no feedback about correctness or incorrectness, nor guidance or suggestions about what approaches to take. There were a total of six PSAs, forming two sets of three problems:

The "non-prime" set: PSA A, the shirts problem,
PSA B, the party tables problem,
PSA C, the dice game problem;

The "prime" set: PSA A', the circus problem,
PSA B', the clocks problem,
PSA C', the spinner game problem.

One problem in each set of three is analogous to one problem in the other set (A with A', B with B', and C with C').

Each child saw all six PSAs, one set of three at the pretest and the other set of three at the posttest. As we have already explained in Volume I, half the children saw the prime set at the pretest and the non-primes at the posttest, and the other children saw the non-prime set first and the prime set at the posttest. At each testing session the subjects did PSA C (or C') and then PSA B (or B') on the first day, and PSA A (or A') on the second day. (Note: Hereafter we will use the notation "PSA A*" to mean "PSA A or A'," "PSA B*" to mean "PSA B or B'," and "PSA C*" to mean "PSA C or PSA C'.")

One Set of Problem-Solving Activities: The Non-Primes

The following is a description of the non-prime set of the PSAs. As noted above, the prime set is very similar; a description of the correspondences within pairs of PSAs are explicitly identified in Appendix II.D.

PSA A: The Subject (S) is given four different-colored cardboard rectangles (2.5 cm by 15 cm) and a cardboard outline of a shirt with four spaces into which the rectangles can fit. S is told that a shirt designer wants to make as many different shirts as he or she can using the four colored rectangles as stripes. The Researcher (R) places the four stripes in one order, and asks S to make a different order. Then R leaves S. When S has made as many orders as possible (or when a predetermined time has been exceeded), R returns and discusses what S has done. (Time allowed: approximately 15 minutes. All times listed include discussion between S and R.) (There are 24 ways in which the stripes can be ordered.)

PSA B: First, R makes sure that S knows what "twice as many" and "optional" mean. Then S is shown 26 cardboard tags (5.5 cm by 5.5 cm). Each bears the name of a guest who has been invited to a party, and the person's age (the ages are 36, 37, 37, 40, 41, 43, 45, 48, 49, 52, 53, 54, 56, 56, 57, 58, 59, 62, 63, 64, 65, 66, 66, 67, 67, and 68). S is told that the task is to assign each person to one of three tables by making three piles of cards. There is to be a table for the older guests, one for people with ages in the middle, and one for younger people. Everyone at the older table must be older than every person at any other table, and everyone at the younger table must be younger than everyone at any other table. Further, each table must have at least five guests. Finally, an optional condition is described: if S wants to, he or she can make one table have exactly twice as many guests as one of the other tables. The directions are repeated, and a written summary of the instructions is provided. Once the S understands the problem, R leaves the table, returning when called back to discuss what S has done. (Time allowed: approximately 15 minutes.) (There are 18 distinct solutions to this problem; for none of these solutions does a break between tables correspond to a break between decades. That is, whenever M and N are consecutive cards at different tables, then M and N are in the same decade.)

PSA C: The S is told that the owner of a game factory, Dr. Game, was recently upset to discover that his archenemy, Mr. Enemy, had broken into the factory and had changed some of the games in some way. One such game is shown to the S. It consists of an octahedral die, labeled alternately 5 and 8; a cubical die labeled with 2, 3 and 4 on pairs of opposite faces; a coin labeled "+" on one side and "x" on the other; nine plastic chips; and two cut-out foam-core players, colored Red and Purple. The Red player wears a sign saying "less than 10" and the Purple player wears a sign saying "10 or more". The mechanics of the game are explained: a dice-thrower (not further explained or identified) rolls both dice and flips the coin. Then the addition or multiplication is performed. If the answer is less than 10, then the Red player gets one chip; if the answer is ten or more, then the Purple player gets one chip. The player with more chips at the end of nine throws of the dice wins the game. The R repeats that there is something wrong with the game, and that S has been hired to find out what is wrong. R then leaves S to work alone. When S calls R back, S's findings are discussed. R then tells S that S has been hired to fix the game. (If S feels that there is nothing wrong with the game, then the discussion of PSA C ends here.) Again S is left alone. When R is called back for the second time S's method of fixing the game is discussed. (Time allowed: approximately 20 minutes to determine what is wrong with the game, and 20 minutes to fix the game.) (What's wrong is that the game is unfair. The probability of Purple's winning any chip is 0.75, and the probability of Purple's winning the whole game is more than 0.95. There are many ways to fix the game -- e.g., by changing the numbers on the dice or the operations on the coin, or by changing the cut-off score of 10, or the number of chips awarded to Red when the sum or product falls below the cut-off score; or through some combination of these.) (This problem is loosely based on a lesson from the Comprehensive School Mathematics Program (CEMREL, 1979).)

Criteria for the construction of the PSAs. The PSAs were designed with several considerations in mind, each of which is more fully discussed below. These considerations concern the PSAs' relation to the show, their relation to each other, and their general characteristics as problems.

With regard to the PSAs' relations to the series itself, we wanted the PSAs to be:

(1) mathematically rich, nonroutine problems that, as a group, incorporate ideas from several subgoals of Goal III (what Goldin & McClintock (1984) would refer to as content variables);

(2) amenable to the use of a variety of problem-solving actions and heuristics delineated in the statement of Goal II (heuristic process variables); and

(3) reflective of the spirit of the series in some sense (context variable).

With regard to the PSAs' relations to each other, we wanted the PSAs as a group:

(4) to span a range of difficulty and complexity (which relates to all of the kinds of variables);

(5) to be structurally similar and of similar difficulty within pairs (since the problems were going to be used in pretests and posttests) without being identical (structural variables); and

(6) mathematically connected in some way (content variables).

As general characteristics, we wanted the PSAs to be:

(7) challenging even to more capable students; yet

(8) accessible to everyone in the sense that all students could understand the statements of the problems and at least begin to grapple with them. (These last two characteristics pertain to relations between task variables and variations among the subjects who are doing the PSAs.)

Further details about these considerations are provided below, numbered in the same order.

With regard to the PSAs' relations to the series itself:

(1) As a group, the PSAs draw mathematical content from several of the eight Goal III subgoals, including ones not emphasized in the typical elementary school curriculum. (See Appendix I.A, of Volume I, for a statement of Goal III.) In particular, combinatorics and probability are featured in PSAs A* and C*. While geometry is not the principal theme of any of the PSAs, there are geometric aspects in all of them -- the spatial order of the objects in PSA A*, the potential for rectangular arrays in PSA B*, and the structure of the polyhedral dice, spinners, and other equipment in PSA C*. We were not interested, however, in comparing the difficulties of tasks as a function of their mathematical content per se; that is, we made no attempt to design the tasks in such a way to allow a comparison of the difficulty of probability ideas, for example, with that of combinatorics.

Table 3.1 summarizes the Goal III subgoals that we identified as being ones that might reasonably be associated with each PSA most directly; although not all of the subgoals listed are strictly necessary to obtain a solution to the PSAs, and others might well be used by some subjects.

Table 3.1

Goal III Subgoals Associated With Each PSA

PSA	Goal III Subgoals	
A*	A1 --	Whole number counting
	B1 --	Basic operations
	E3 --	Combinatorics, systematic enumeration of cases
B*	A2 --	Place value for organizing cards by decades
	B1 --	Basic operations
	B5 --	Ratio (for the optional condition)
	D1 --	Ordering
C*	B1 --	Basic operations
	D1 --	Order relations
	E1 --	Combinatorics, multiplication principle
	F1 --	Probability, basic quantification
	F3 --	Independence
	F5 --	Data collection and analysis
	G6 --	Geometrical objects

The PSAs are nonroutine not only in their content, but also in their format. Unlike many problems in typical school mathematics programs, the PSAs require extended effort, and the subjects are given ample opportunity to work alone on them. Additionally, all of them have a manipulative component that is very different from the common school experience at the fifth-grade level. (The mode of presentation -- manipulative versus purely verbal format -- is a context variable.)

(2) Each one of the PSAs can be approached from a variety of directions, as described in the Goal II subgoals. This variety is reflected in the number of Goal II subgoals (actually, subsubgoals) that we identified as ones that are necessary to use, or might be particularly helpful, in working on the PSAs. (This determination was reached through pilot-test results and analysis of the content of the problems. Certainly other subgoals might be used occasionally, but they seemed less likely a priori.) Table 3.2 presents these subgoals.[10] The relation-

Table 3.2

The Goal II Subgoals Involved in Each PSA

PSA	Goal II Subgoals
A*	Necessary B4x -- Some kind of calculation or counting Potentially helpful B2 -- Estimating the number of orders B6 -- Trial and error in generating orders C1x -- Listing the orders C2 -- Simplifying into 3-object problem C3x -- Noting patterns in arrangements D3 -- Finding another way to solve it

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[10] An indication of the PSAs' variety as perceived by the subjects themselves is the number of Goal II subgoals, out of a possible 17, that were actually used fairly often. It turned out that in PSAs A* and C*, 11 of the 17 subgoals were used by at least ten subjects, and that 8 of the 17 were used by at least ten subjects in PSA B*.

Table 3.2 (cont.)

B* Necessary
B4x -- Calculation (twice as many)
B4y -- Manipulation of the cards

Potentially helpful
B3 -- Looking through cards
B6 -- Successive adjusting of pile sizes
C1y -- Using objects to stand for tables/clocks
C2 -- Making other groupings, e.g., decades
C3y -- Rejecting extraneous data (e.g., names)
C4 -- Changing from decade organization
D2 -- Finding another of the 18 solutions

C* Necessary
B4x -- Calculations to play game,
or to find numbers of outcomes;
B4y -- Manipulations in playing game
B5 -- Probabilistic thinking

Potentially helpful
B2 -- Estimation of probabilities
B3 -- Gathering information through playing game
B6 -- Adjusting of split between winning numbers
C1x -- Charting possible sums and products,
or listing outcomes as they occur
C2 -- Simplifying by keeping one device fixed
C3x -- Noting patterns on probability devices,
or in numerical results
C4 -- Abandoning one fixing method for another
D1 -- Checking, by replaying game, to see if fair
D2 -- Making game fair in another way

ship between the individual problems (A*, B*, and C*) and the problem-solving actions and heuristics that they tended to elicit is examined in detail in Chapter 6.

(3) The six problems that were used reflect the spirit of **SQUARE ONE TV** in that each PSA is built around some sort of story as an underlying context. It is easy to imagine that any one of the PSAs could have been an assignment for one of the series' writers to produce a **SQUARE ONE TV** segment for broadcast.

In fact, finding the number of orders in which four distinct objects can be placed is the basis of a Season I segment, although that segment was _not_ included in the 30 programs used as the treatment. The same combinatorial idea, restricted to the number of orders of _three_ distinct objects, did appear in a segment shown to the experimental group, however. (There is no similar connection between any of the segments in the 30 programs of the treatment and either PSA B* or PSA C*.)

With regard to the PSAs' relations with each other:

(4) Clearly the problems are of _varying complexity_, increasing from PSA A* to PSA C*. This is evident simply from the relative lengths of their descriptions in the preceding section (or in Appendix II.D, which shows the parallel structure of the primes and non-primes).

The increasing complexity is also evident if one considers the number of distinct objects and relations that one must keep track of in attempting to solve the problem. In PSA A* there are a relatively small number of objects to arrange within a concrete framework (the shirt or the evening's program), even though the objects of real interest are distinct orders, rather than the stripes or performers themselves. PSA B* imposes several conditions on the arrangement of cards, and the objects around which the cards are grouped (the tables or shelves) must be created in the subject's imagination. PSA C* is even more complicated, with two kinds of probability devices, players, chips, signs, etc., and the problem itself is much less well defined for the child than in the case of PSA A* or PSA B*. The subjects apparently agreed that PSA C* is the most difficult, and PSA A* the least.[11]

(5) Problems within the same pair are _structurally similar_. Each problem in the non-prime set is paired with one problem in the prime set (A with A', B with B' and C with C'). The links between the problems in each pair are explicitly indicated in Appendix II.D. As that

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[11] Perceived difficulty was assessed in the second day's interview, when the subjects were asked which PSA was the most challenging. There were 96 opportunities to make such an assessment (48 subjects, each with a pretest and a posttest). Of those 96 opportunities, A* was said to be most challenging 8 times; B* 18 times; and C* 66 times. On 4 occasions, the question was not asked or there was no response.

Appendix shows, the connection between PSAs is strongest for PSAs A and A', which are isomorphic, and somewhat weaker for PSAs C and C'.

Nonetheless, as the pilot testing had suggested, the PSAs in each pair are of essentially equal difficulty. This finding was confirmed in the main study; see Chapter 5.

(6) There are two principal mathematical connections among PSAs A*, B* and C*, which, roughly speaking, are these: PSA C* and PSA A* share the combinatorial idea of multiplying to find the total number of possibilities; and PSA C* and PSA B* share the idea of adjusting a cut-off number to create sets with desired properties. There is also a combinatorial relationship between PSA A* and PSA B*; a full discussion of all of these relationships appears in Appendix II.E.

With regard to more general characteristics of the PSAs:

(7) The PSAs are challenging even to the more capable students. On a measure of the mathematical sophistication and completeness of solutions to the PSAs (the M-score, discussed in detail in Chapter 4, on the coding systems), only two subjects achieved the highest possible score on PSA A*, while none achieved the highest on PSA C*. Thirteen subjects achieved the highest possible score on either PSA B or PSA B'; possible reasons for this are discussed later in this report. (Of course, this is not to imply that children who received high, or even the highest, M-scores on a particular PSA were not challenged by it.)

(8) Although the PSAs are challenging to most of the subjects, they are accessible at some level to everyone. Every subject received a positive M-score for PSA A* and PSA B*; and while there were six subjects who could find nothing wrong with the game in PSA C*, all of them made some inroads into the problem through some kinds of problem-solving actions or heuristics.

CHAPTER 4

THE CODING SYSTEMS

Introduction

The purpose of this chapter is to describe the coding systems that were used in the study and how they were developed and applied.

Conceptual Framework for the Coding Systems

Using the goals of the series. The coding systems that were created were a natural outgrowth of one of the major premises around which the entire study was built: our desire for alignment between the goals of **SQUARE ONE TV** and the instruments used to assess viewers' problem solving. We will say considerably more about this later, but, in essence, we used the goals of the series, particularly Goal II, as the basis of our measure of problem-solving behavior.

Using complex problems. Because problem solving, as presented on **SQUARE ONE TV**, is a complex enterprise, any reasonable assessment of what viewers derive from the series would have to involve their grappling with similarly complex problems. As we have already described in Chapters 2 and 3, we created suitable assessment instruments that involved task-based interviews with individual children.

This choice of methodology raises a number of interrelated questions that must be addressed before proceeding further: (a) How does one know that the behavior being coded actually reflects what the subject is thinking? (b) How can a coding system condense or summarize information obtained from an interview in a way that is useful? (c) What kind of information is lost in going from interview data to some kind of summary data? We will discuss each of these questions below.

Does behavior reflect thinking? This is a very difficult question, one that has been the subject of much debate in the literature (Ericsson & Simon, 1984; Kilpatrick, 1978; Schoenfeld, 1982). In the past, many researchers have relied on a "think-aloud" technique (e.g., Kilpatrick, 1967), usually with subjects older than ours. Indeed, as we indicated in Chapter 2, our experience in the pilot-test phase convinced us that many fifth-grade subjects found it very difficult to think aloud consistently, without fairly constant prodding from the interviewer. As a result, we settled on the hybrid strategy described there: videotaping individuals working alone, immediately followed by an interviewer's asking the subject to describe his or her actions and thinking, probing with follow-up questions as needed, all in a neutral context explicitly designed to encourage honest and thoughtful retrospection. Of course one can always wonder if what a subject said was different from what he or she was "really" thinking, or even if the subject knew what he or she was thinking. Regardless of the methodology, no one can guarantee a perfect match between thought and deed; all we can do is arrange circumstances to maximize the likelihood that a subject's description of his or her own problem-solving behavior is an accurate one.

Sample size vs. data richness. The basic issue here is a trade-off between the number of children who can be interviewed and the depth of information that can be obtained. That is, given a fixed level of resources available for a research project, one must balance the need for representativeness of the sample against the need for sufficient detail about each child's thinking. Our approach in this study was to determine first the length of time required to get the level of detail we required, and then to determine the largest number of subjects who could participate within the time frame available.

How can coding summarize a set of interviews? A related issue concerns the competing demands of sample size and the need to create a meaningful picture of what the subjects are saying and thinking. The larger the number of subjects, the more difficult it may be to obtain a comprehensible summary of the data. This, of course, is the motivation behind the development of a coding system. Ideally, the system should quantify the subjects' behaviors in a way that reflects the goals of **SQUARE ONE TV** while maintaining the integrity of the children's problem-solving performance. This quantified information, in turn, can be analyzed and summarized statistically, without losing its connection with the series itself.

What information is lost in coding? Clearly no coding system can preserve all the information in an interview, or else it would be equivalent to the interview itself; of necessity, then, something must be lost. As we have already noted, our interview coding system does not record sequences of behaviors (as opposed to the behaviors themselves). Nor does the problem-solving coding system directly record any indication of subject affect. (See Volume III for discussion of children's attitudes toward mathematics and problem solving). The remainder of this chapter is devoted to explaining how the coding systems work. The reader can obtain a clear picture of what information they do capture by reading the annotated interview at the end of this chapter and the additional interviews in Appendix II.F.

Overview of the Coding Scheme

We measured subjects' performance on the Problem-Solving Activities with two separate scores, which we call the P-score and the M-score. The P-score ("P" for "problem-solving") is a measure of the extent to which the subject uses the problem-solving actions and heuristics that are described in the subsubgoals of Goal II. (See Figure 1.2 of Chapter 1.) The P-score summarizes the number and variety of actions and heuristics that the child uses.

The M-score ("M" for "mathematical") is a measure of the mathematical sophistication and completeness of the subject's solution of the PSA. The rationale for using two different scores is discussed below.

Descriptions of P-score and M-score

We turn now to a more detailed description of what the P-scores and M-scores are and how they were derived.

The P-score. Our method of measuring the extent to which subjects use problem-solving actions and heuristics is a direct application of NCTM's (1989) idea of alignment: The system for assigning P-scores is essentially derived from the subgoals of **SQUARE ONE TV**'s Goal II.

The chain of reasoning is this: **SQUARE ONE TV** segments portray characters engaging in mathematical problem solving in a variety of situations. These segments have been coded on the basis of the Goal II subgoals that they depict. Children who watch the series repeatedly may, as a result, be more likely to use behaviors that would be categorized under these subgoals. (This use might be conscious or unconscious.) To see if in fact this is true, it is reasonable to use this same system of subgoals to categorize the children's behavior.

Note that one would not necessarily expect children to mimic the behavior shown by **SQUARE ONE TV** characters exactly. The actors are carefully scripted, and what they do is often designed to illustrate problem-solving strategies as clearly as possible. Further, two of the PSAs (B* and C*) are very different from any particular segment that was included in the six weeks of programs, and the third one is only partly similar to one of the segments. Thus, the particular behaviors that children use in working on the PSAs will not be exactly the same as those that they have seen during the treatment. Nonetheless, we can use the same subgoal system to categorize the children's behavior even if that behavior is not precisely what one would see modeled in the series itself.

So, with some minor modifications (described below), we simply took the full statement of Goal II and used it to code the children's problem solving as if their performances were segments on the program. Our experience during the pilot-testing phase showed this to be a reasonable approach; that is, the problem-solving behaviors that were elicited in the pilot-test phase were ones that could be categorized according to Goal II subgoals.

he modifications that we made to the series' Goal II scheme consisted of (a) not using the Problem Formulation subgoal (IIA) at all; and (b) slightly reorganizing some of the details at the subsubgoal level. A few explanatory comments on each of these changes are as follows:

(a) The Problem Formulation subgoal (IIA) was not used because in each PSA the problem was posed for the subject. Of course we realize that "problem formulation" refers to the problem solver's formulation of the problem, not to the poser's formulation. But because the problem was explicitly presented to each child, it seemed very likely that every child would formulate the problem in some way, and thus get credit for Problem Formulation (IIA) if it were available; and in fact that was the case. Occasionally, a child would interpret a problem in some very unusual, nonmathematical way; this affected the mathematical completeness and sophistication of his or her solution and hence was captured in the M-score for that PSA.

Occasionally, too, a child would generate a new problem in the course of working on a PSA. This was captured in "look for, or extend to, related problems," which is subsubgoal IID4.

Finally, we note that a few children spontaneously assessed the difficulty of a problem (IIA3), but their comments seemed more affective in nature than directly related to the mechanics of solving the problem at hand. Issues related to affect are explored more fully in Volume III, on the Attitude Interviews.

(b) The minor reorganization of subsubgoals involved splitting Calculate or Manipulate (IIB4) into Calculate (IIB4x) and Manipulate (IIB4y); and collapsing certain sets of subsubgoals (e.g., IIC1a (use a scale model) through IIC1d (use a graph) were collapsed into IIC1x (use a diagram, etc.)). The two central columns in Table 4.1 show how the Goal II subsubgoal structure appeared in **SQUARE ONE TV** and how it was modified for this study.

For each subject on each PSA, one or more coders looked at the entire videotape and verbatim transcript of the child's behavior, including the discussion between the interviewer and the child. The coders were blind as to whether the child was a viewer or nonviewer, just

Table 4.1

A Comparison of the Series'
and the Study's Interpretation of Goal II

GOAL II SUBSUBGOALS FOR:	THE SERIES	THE STUDY	
IIA. PROBLEM FORMULATION			
Recog., state a problem	IIA1		Not used at all, since interviewer posed the problems
Assess value of solving	IIA2		
Assess possib. of solving	IIA3		
IIB. PROBLEM TREATMENT			
Recall information	IIB1	IIB1	Same as SQUARE ONE TV
Estimate, approximate	IIB2	IIB2	
Gather data	IIB3	IIB3	
Calculate or manipulate	IIB4	IIB4x IIB4y	Calculate Manipulate
Consider probabilities	IIB5	IIB5	Same as SQUARE ONE TV
Trial & err; guess & chk	IIB6	IIB6	
IIC. HEURISTICS			
Scale model	IIC1 a	IIC1x	Diagram, etc.
Diagram	b		
Table, chart	c		
Graph	d		
Use objects; act out	e	IIC1y	Use objects; act out
Transform problem	IIC2	IIC2	Transform problem
Look for patterns	IIC3 a	IIC3x	Patterns
Look for missing info	b	IIC3y	Missing info; pert. vs. extr.
Pertinent vs. extraneous	c		
Change point of view	IIC4 a	IIC4	Reapproach problem
Generate new hypotheses	b		
IID. PROBLEM FOLLOW-UP			
Reasonableness	IID1	IID1	Same as SQUARE ONE TV
Alternative solutions	IID2	IID2	
Alternative ways to solve	IID3	IID3	
Extend to related probs.	IID4	IID4	

as the interviewers had been.[12] The coder awarded points (a maximum of two) for each instance of the child's use of a problem-solving action or heuristic. This was uniformly done in accordance with procedures laid out in a detailed P-Score Coding Guide (included in this report as Appendix II.G). The reader is referred to that Appendix for the complete details; a few general points are highlighted here:

1. With very few exceptions, behaviors were awarded points only if they were explicitly displayed, either through the child's actual doing of something or else through his or her subsequent description of it in the discussion with the interviewer.[13] Coder inferences were kept to a minimum.

2. Only the child's first use of a problem-solving action or heuristic for a particular purpose was awarded points. For example, a subject might draw a chart and get a score for IIC1x. If he or she later drew a different kind of chart for a different reason, then more points would be awarded under IIC1x. But if the subject drew a different version of the same chart for the same purpose, then no new points would be awarded.

3. The same general scheme was applied to all the PSAs.

4. The P-score coding was essentially open-ended in the sense that it set no limit on the number and variety of problem-solving actions and heuristics that a subject might use.

5. Any points awarded, together with any relevant commentary or reference to other parts of the interview, were recorded directly on the transcript. In addition, these data, plus

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[12] Note that coders were not blind to whether the interview was from the pretest or the posttest because of the slight differences between the introductions to the interviews. This is of little importance, however, because the viewer/nonviewer status was unknown. Also, one child mentioned **SQUARE ONE TV** in a PSA A interview, but this happened to be the last interview conducted at that school (so that subsequent interviews could not be affected); also, it was the last interview coded.

[13] An example of an exception: a child was awarded points for Calculation (IIB4x) if he or she played a whole game in PSA C*, under the assumption that the only way to play the game is to perform the additions and multiplications with the numbers on the spinners or dice. Ordinarily, of course, a child who did this would mention the calculations during the subsequent discussion with the interviewer, thus acquiring the points anyway. For details, see Appendix II.G.

a summary comment describing each problem-solving action and heuristic, were recorded on a separate coding form. An example is shown in Figure 4.1, which appears after the next section on the M-score.

The M-score. The M-score is a measure of the child's mathematical success with a PSA. It is derived from two sources: (a) a mathematical analysis of what the particular PSA involves, and (b) the range of mathematical ideas expressed by subjects in the pilot-testing phase[14]. Coding was again based on examination of the videotape and transcript, with guidance provided by the M-score Coding Guide (which is Appendix II.H).

A set of comments that parallels those made above about the P-scores can be made about the M-score:

1. Only behavior that was explicitly present was awarded points.

2. Since the M-score reflected how far the child progressed with the PSA, it was a reflection of the child's most advanced, final thinking on the PSA. Unlike the P-score, it mirrored the ultimate destination of the problem solving, rather than the actions and heuristics used in the process of problem solving.

3. Unlike the P-score coding, the M-score scheme did not apply generally to all the PSAs because they were different from each other mathematically. There was one set of scores for PSA A*, one for PSA B* and a third one for PSA C*.

4. Unlike the P-score, the M-score was not open-ended: a top score was awarded for a full and complete solution to the PSA; these maximal scores were different for PSAs A*, B*, and C*. If a child suggested more than one solution, the most sophisticated one was used as the basis for assigning an M-score.

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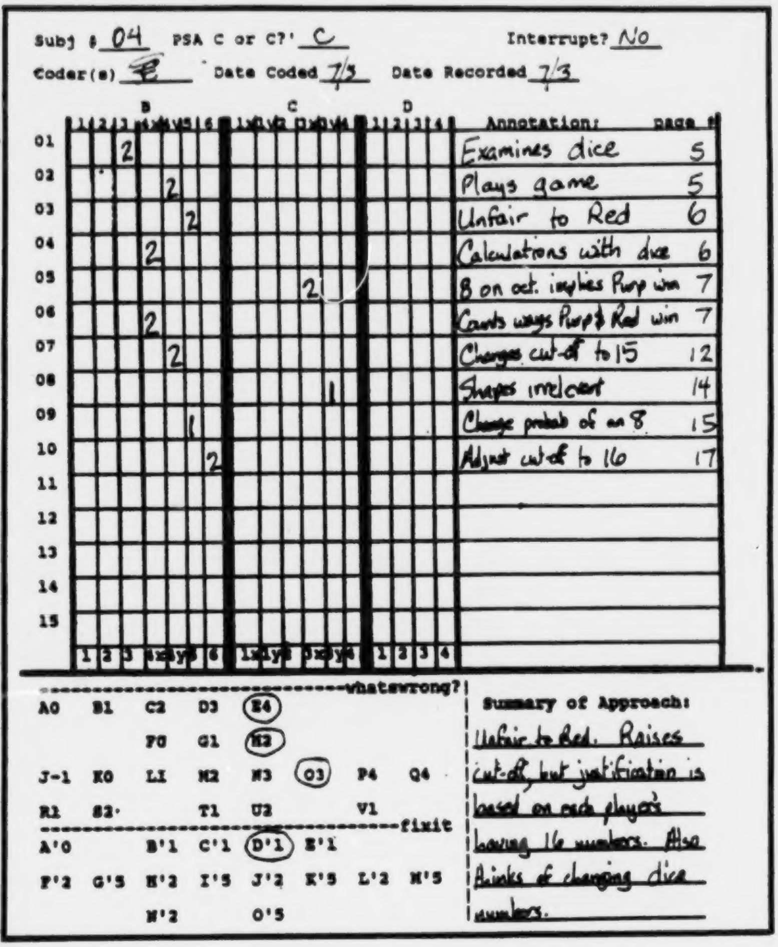
[14] Thus, the measure of mathematical sophistication used here is partly a function of the overall mathematical sophistication of the fifth-grade subjects. A much more sophisticated subject pool (e.g., mathematics graduate students) would probably suggest insights, connections, extensions or generalizations that are not included in the M-score system. The scoring system is inclusive enough, however, to cover not only the most sophisticated responses from children in both the pilot study and main study, but also what is, in fact, necessary for full solutions to the PSAs as presented.

5. Each subject's M-score for a particular PSA was recorded directly on the coding form. The format for recording this differed from one PSA to another; that is, there was one form for PSA A*, another for PSA B*, and a third one for PSA C*. At the bottom of the coding form was space for an overall summary of the child's performance. Figure 4.1 shows an example of a coding form for PSA C* (i.e., to be used for either PSA C or PSA C').

Figure 4.1

A Completed Coding Form for PSA C*



Subj # 04 PSA C or C?' C Interrupt? No

Coder(s) [illegible] Date Coded 7/3 Date Recorded 7/3

	B							C						D				Annotation:	page #
	1	2	3	4x	4y	5	6	1x	1y	2	3x	3y	4	1	2	3	4		
01			2															Examines dice	5
02					2													Plays game	5
03						2												Unfair to Red	6
04				2														Calculations with dice	6
05											2							8 on oct. implies Purp win	7
06				2														Counts ways Purp & Red win	7
07					2													Changes cut-off to 15	12
08												1						Shapes irrelevant	14
09						1												Change probab of an 8	15
10							2											Adjust cut-off to 16	17
11																			
12																			
13																			
14																			
15																			
	1	2	3	4x	4y	5	6	1x	1y	2	3x	3y	4	1	2	3	4		

---------------------------------whatswrong?

A0 B1 C2 D3 [✓ E4]

F0 G1 [✓ H2]

J-1 K0 L1 M2 N3 [✓ O3] P4 Q4

R1 S2 T1 U2 V1

---------------------------------fixit

A'0 B'1 C'1 [✓ D'1] E'1

F'2 G'5 H'2 I'5 J'2 K'5 L'2 M'5

N'2 O'5

Summary of Approach:

Unfair to Red. Raises cut-off, but justification is based on each player's having 16 numbers. Also thinks of changing dice numbers.

The column headings are Goal II subgoals.

This section of the form is for recording the P-score. This section is the same for every PSA. The 1's and 2's represent numbers of P-score points awarded.

This section is used for recording the M-score for PSA C or C'; the letters and numbers are keyed to the M-Score Coding Guide.

Relation between P-scores and M-scores. Can the P-scores and M-scores vary independently? The answer is affirmative, as we can see by considering, for example, two subjects working on PSA C, the dice game. One subject might engage in several problem-solving actions or heuristics in attacking the problem. He or she might play the game, examine the game pieces, make a list of the numbers on the dice, make another list of the outcomes of several tosses of the coin, and finally come to some vague conclusion that there's something wrong with the octahedral die. This would result in a relatively high P-score (due to the large number of actions and heuristics used) but a low M-score (reflecting the lack of sophistication of the solution).

Another subject, confronted with the same problem, might simply look at all the numbers on the dice and immediately conclude that the game is unfair through a more or less complete analysis of the possible sums and products. In contrast to the first subject, this would result in a low P-score and a high M-score.

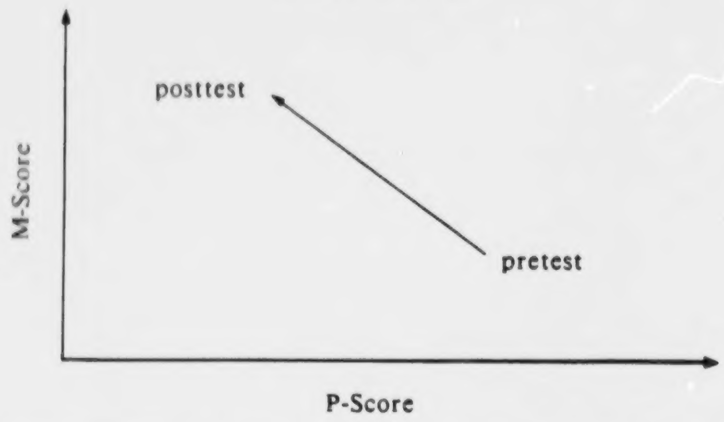
Since P-scores and M-scores are evidently conceptually independent, one can represent a subject's P-score and M-score on a particular PSA by a single point in a P-M plane. The change in a subject's P-score and M-score from pretest to posttest can then be shown as a vector in the P-M plane. Figure 4.2 illustrates how a pretest-to-posttest change involving an increasing M-score and decreasing P-score would be depicted.

One would expect, of course, that for a fairly homogeneous population of subjects there will be a positive correlation between the two scores. That is, other things being equal, the more problem-solving actions and heuristics one tries, the more likely one is to find a solution to the problem. After all, this is what leads us to describe the actions and heuristics with the adjective "problem-solving" in the first place. (The actual correlations are presented in Chapter 5 of this volume.)

Despite a likely correlation between the two scores, we felt both were needed, particularly since we were looking at changes in the children's performance over time. We

Figure 4.2

An Example of Pretest-to-Posttest Change in the P-M Plane



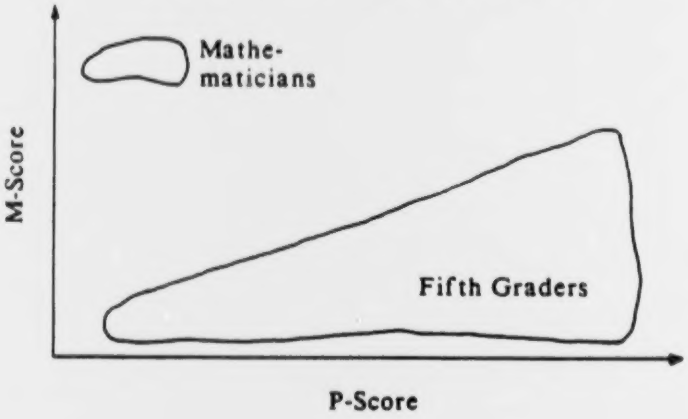
wanted to see if in fact greater P-scores were associated with greater M-scores; it was also entirely conceivable that some of the subjects, on their second exposure to the testing, might reach a better solution to a problem, but engage in fewer problem-solving actions or heuristics (i.e., obtain a higher M-score and a lower P-score, as illustrated in Figure 4.2). Others might try more of the problem-solving actions and heuristics the second time but get no farther into the problem mathematically. Only through an analysis of both scales would a full picture of the subjects' problem-solving performance over time emerge.

Given this two-dimensional way of quantifying problem solving, it is reasonable to ask what kinds of changes over time should be considered "improvements." Certainly any increase in M-score should be deemed an improvement, but is an increase in P-score always desirable?

In addressing this question it is crucial to keep the subject pool -- fifth graders -- in mind. As we indicated in Chapters 2 and 3, on pilot testing and instrument development, the PSAs were created to be genuine problems for this particular age group, requiring from virtually all students some explicit problem-solving actions or heuristics for solution. In contrast, for a group of mathematicians the PSAs would not be "problems" at all. In particular, they would find it unnecessary to engage in the kinds of behaviors that fifth graders often find helpful -- moving objects around in PSAs A* or B*, for example, or playing the game in PSA C*. Generally, then, their M-scores would be very high and their P-scores low. Figure 4.3 illustrates schematically where the two populations' scores on some hypothetical PSA might be located in the P-M plane. As the figure suggests, one would expect the region of mathematicians' scores to be generally above and to the left of the fifth graders' region, and also to occupy a smaller area.

Figure 4.3

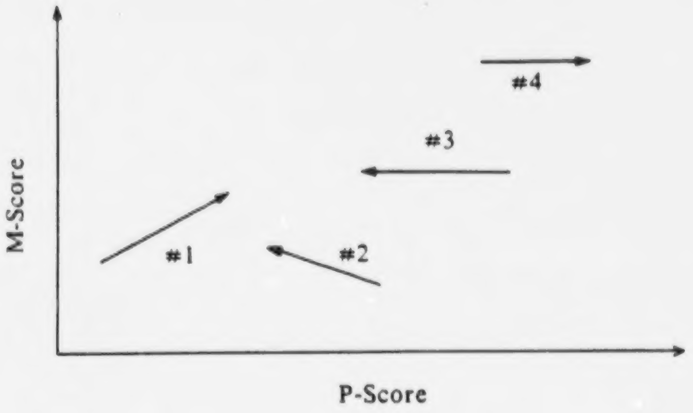
The Locations of Mathematicians' and
Fifth Graders' Scores in the P-M Plane



Now consider how a student's P-score and M-score might change from pretest to posttest. In Figure 4.4, each arrow in the P-M plane represents a change from pretest to posttest. Arrow #1, for example shows that a subject's P-score and M-score have both increased from pretest to posttest. We have already claimed that any increase in M-score should be regarded positively; conversely, a decrease in M-score should be seen as negative. Sometimes (as in the case of arrow #2) an increase in M-score might be accompanied by a decrease in P-score. The

Figure 4.4

Some Hypothetical Changes in Four Individuals' P- and M-Scores, Shown in the P-M Plane



change shown by arrows #2 or #3 could be interpreted as an increase in "efficiency" -- that is, the same or greater M-score performance with less problem-solving activity.[15] The only real question is what value to place on a change in performance like the one shown by arrow #4, for which the M-score is constant from pretest to posttest, with an increase in P-score.

We have adopted the general position that the only circumstance in which an increase in P-score with a constant M-score might not be desirable is when the M-score (both pretest and posttest) is at the maximum for that PSA, as in arrow #4 in Figure 4.4. Whenever an M-score is less than maximal, then more problem-solving behavior (i.e., an increase in P-score) provides at least an opportunity for a more sophisticated solution to be discovered. If the M-score is already at a maximum, then the use of additional problem-solving behavior may serve no purpose. We caution, though, that one's interpretations of an individual subject's problem-solving performance should be guided by more than the just the numerical P- and M-scores: one should examine the specific behaviors that are contributing to those scores.

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[15] As it turned out, there was little need to be concerned with the issue of "efficiency," because few children showed an M-score increase and a P-score decrease. Indeed, taking an extreme case, one would expect the increase in efficiency that would be shown by a move in Figure 4.3 from the "fifth graders" portion of the P-M plane to the "mathematicians" portion to take place over a period of years, not weeks.

Other Scores: Row-Score and Column-Score

Two other scores were calculated for each subject and PSA, both of which are derived from the P-score coding and are closely related to the P-score.

The Row-score is the number of rows on the coding form that were used to record the subject's problem-solving actions and heuristics. Since each action and heuristic took one row, the Row-score is thus a measure of the sheer number of discrete problem-solving behaviors that the subject engaged in. The Row-score for the person in Figure 4.1 is 10 because 10 rows were used for coding.

The Column-score is the number of distinct columns that were used to record the subject's actions and heuristics. Because each subgoal was assigned to a separate column, the Column-score is a measure of the variety of the child's problem-solving behavior. The Column-score for the person in Figure 4.1 is 7 because seven columns were used.

It is evident from the definitions of these scores that they must be highly correlated, and indeed the correlations among the P-, Row-, and Column-scores all exceed 0.90 (see Chapter 5 and Appendix II.K). The reason for introducing the Row-score and the Column-score at all is that they are measures of problem-solving behavior that do not depend on any particular scheme of awarding points for those behaviors. If, for example, one were to devise a more fine-grained system in which behaviors were awarded from 0 through 5 points (rather than our 0 through 2), then different P-scores would result, but the Row- and Column-scores would remain the same.

Determining Interrater Reliability

The coding guides (Appendix II.G and II.H) were drafted on the basis of our experience with the pilot-test subjects and, for the P-score, how their behaviors related to the Goal II subgoals. During the pilot-test phase the minor reorganization of the subgoals depicted in Table 4.1 was carried out, and we codified the mathematical content of the PSAs for the M-score guide. Dozens of specific examples of behaviors that fall under particular subgoals were listed to provide guidance to coders so as to encourage uniformity. Some of these specific examples were taken directly from pilot-test subjects and are identified as such in the guides.

Two coders independently coded some of the pilot-test subjects' videotapes and transcripts. These codings were discussed and the guides revised and clarified so that ambiguities were minimized.

This process was repeated several times over the early part of the study, using data from the pilot test and the main study, with successive refinements of the wording and examples in the guides. In all, there were 11 revisions of the P-score coding guide and five revisions of the M-score guide.

Interrater reliability was assessed using transcripts coded by two raters. Twenty transcripts were used, each coded for the three PSAs. The reliability set included a representative sample of boys, girls, viewers and nonviewers, from all the schools involved in the study. In summary, the reliability of the coders' judgments, at the end of the guide refinement process, was satisfactorily high.[16] Once that determination had been made, one of the coders continued to code the remaining tapes and transcripts.[17] Throughout this process, coders were blind to the viewer/nonviewer status of the children.

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[16] For P-scores the mean percent of agreement was 92%, with a kappa statistic of .51. For M-scores, raters' scores were correlated at an average of 0.96. These reliability data are presented in Appendix II.I.

[17] Videotapes, transcripts and codesheets are on file; readers may contact the authors for further information.

An Example of an Interview

We close this chapter by presenting an example of a transcript of an interview about PSA C'. The lefthand column contains commentary that describes what P-score points were awarded and why. Additional examples of interviews are provided in Appendix II.F.

Student ID 3
PSA C'
DOC: 4096M

Researcher: ...Before we start I'd like to say that what we're interested in is how you think about things that we're going to do today. So, at the end of each thing that I ask you to do, I'm going to ask you a lot of questions about what you did and what you were thinking; so if you could try to remember what you were thinking while you're doing these different things -- that'll be really very helpful to me. And when I ask you the questions at the end, I really do want to know everything you thought -- your good ideas, your bad ideas -- even the ideas that you think don't really matter. They're all important, really, okay?

Student: Okay.

R: Is that clear?

S: Yeah.

R: Do you have any questions?

S: (SHAKES HEAD)

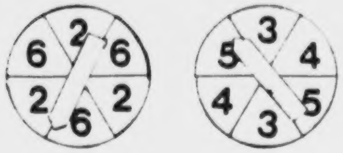
R: No? Let's begin, then. Before I explain what I'd like you to do today, I'm going to give you some background information. The owner of a game factory, Dr. Game, was recently very upset to discover that his factory had been broken into, and his arch-enemy, Mr. Enemy, got into several of his games, and made something wrong with them. He's changed them in some way. And you've been hired to help find out what is wrong with one of the games that Mr. Enemy has changed. I will show you the pieces for the game, and tell you how it works, and then it's your job to find out what it is that's wrong.

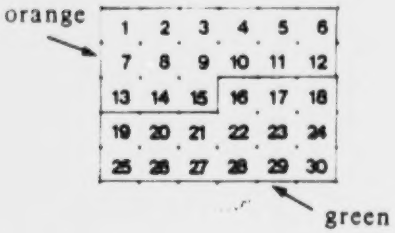
S: Okay.

R: Great! All right, here are two spinners for the game. And here is a coin. Why don't you take a look at those?

...Okay? Got a chance to look at them?

S: (NODS)





R: Okay. Here's the game. There are two players, a green player and an orange player. That's them (INDICATES PLAYERS). And there's also a spinner person. Now, the spinner person spins both of the spinners and gets two numbers, one on this and one on this spinner (SPINS). If it lands on a line, the spinner person just spins it again. Then the spinner person flips the coin, and gets a plus or a times (FLIPS AND SHOWS BOTH SIDES OF COIN). And then the spinner person does whatever is on the coin, either the addition, or the multiplication, and finds the answer here, on this chart (INDICATES THE NUMBER BOARD). If the answer is inside the orange loop -- this orange loop (POINTS) -- then the orange player gets a chip, (MOVES CHIP IN FRONT OF PLAYER) and if the answer is inside the green loop, the green player gets a chip (MOVES CHIP APPROPRIATELY). Now, there are nine chips here (INDICATES CHIPS). Whoever has more chips at the end of nine spins of the spinners wins the game. Those are the rules. Does this make sense so far?

S: Uh huh.

R: Okay. Um, let's try an example. Suppose this spinner comes up with a two (MOVES POINTER SO THAT IT POINTS TO 2), and this spinner comes up with four, (MOVES POINTER OF OTHER SPINNER). And the coin is plus. So, you do the addition, two plus four is six, right? And then you look to find six on the chart.

S: Orange. (POINTS TO ORANGE MAN)

R: Right. Six is in the orange loop, so the orange player gets a chip. (MOVES ON CHIP TO THE ORANGE PLAYER, THEN RETURNS CHIP TO PILE.) All right, let's try another one. Suppose that this spinner shows six, and the other spinner says three. (MOVES POINTERS APPROPRIATELY.) And the coin comes up times. Who would win the chip then?

Even if the child had said explicitly that six times three is eighteen, no points would have been awarded for IIB4x (calculation) because the calculation was done in direct response to the researcher's question.

S: The green one.

R: The green one, right? Because six times three is eighteen and eighteen's in the green loop. Right. Okay. So those are the rules. Remember, there's a bunch of stuff here that you can use if you like to (POINTS TO EQUIPMENT KIT). If you want anything, just help yourself. You may want to use something, you may not want to. That's up to you, okay? And, again, what I said before is that there's something wrong with the game, and you've been hired to find out what is wrong. You should know before you start thinking about this that Dr. Game took a quick glance at the game and was relieved to notice that none of the pieces were missing. But there's still something wrong with the game, all right? Now, I'm going to go over there while you're working on this. Why don't you take a little while to think about this. Don't rush. Take as much time as you like, and if you need something or you need to ask me any questions, please go right ahead. I'm right here.

S: Okay.

R: And then, when you think you've found out what is wrong with the game, let me know and I'll come back, and then we can talk about what you think.

S: Okay.

R: Okay? Good! I'll be over there. (R LEAVES TABLE, MOVES TO OTHER SIDE OF ROOM.)

S WORKS INDEPENDENTLY FOR 3:00 MINUTES:

SLOWLY MOVES THE POINTERS ON BOTH SPINNERS, AS IF TO SEE HOW THEY WORK.

IIB4x (calculate) 2 pts.
Counts the numbers.

COUNTS ALL THE NUMBERS IN THE ORANGE LOOP.

The writing does not merit points for IIC1x (diagram, etc.) here.

GETS PAPER AND PENCIL; WRITES SOMETHING AT TOP OF PAPER (BUT NOT CLEAR WHAT IT IS [and, as it turns out, this is not discussed at all by S later]).

IIB4x (calculate) not awarded again for this; nothing new.
IIB3 (gather info) 2 pts. for examining equipment, but only because it is confirmed later in the interview.

COUNTS THE NUMBERS IN THE GREEN LOOP

STARES INTENTLY AT THE EQUIPMENT, BACK AND FORTH BETWEEN THE SPINNERS AND THE NUMBER BOARD.

S: I found out what's wrong.

R: You found out what's wrong? Okay. What do you think is wrong with the game?

IIB4x (Calculation) 2 pts.
Calculation with spinner numbers (also done later).

S: Because uh with the orange one -- and there's no possible way to get the one (POINTS TO THE NUMERAL 1 INSIDE THE ORANGE LOOP).

R: Okay.

S: For the green, the 16 and the 17 -- he can't get those (POINTS TO THE NUMERALS ON THE BOARD).

R: Okay. And how do you know that?

S: Because there's -- if you multiply the two highest numbers. Or if you multiply by any of 'em, it won't come out to 16 or 17 and, if you add and the numbers ain't big enough. And for one [the number on the board], the numbers [on the spinners] are over [more than] one, so...

R: Okay. And what makes that wrong with the game? What about that?

Not entirely clear what is meant here; S seems to have forgotten the rules of the game. R decides not to pursue at this point.

S: When you're playing somebody, the... When you're playing somebody, you -- you can't -- it [the board] shows these numbers but you really can't get some of 'em [e.g., 16 or 17], it's -- you -- you'd be trying and trying and say you get 'em all, and there's still more chips, and then they'll -- there's only the ones that you can't get on this board left, and then there's no winner.

R: Uh huh. Okay... Um, all right... Well, I was over there while you were doing this. And I want to know what you were doing and what you were thinking. So, what did you do first? And what were you thinking?

S: The first thing I did is I thought that one of 'em [players] would have more numbers [inside that player's loop] than the other, so I counted the numbers.

R: Hm hm.

IIC3y (Pert. vs extraneous) 2 pts. Rejects an extraneous feature.

S: But they had the same amount.

R: Uh huh. Okay.

This and previous comment are pertinent only if S is considering probability (IIB5), but S has not been fully explicit. (Points for IIB5 are awarded later, though.)
Note also that the ease of getting green numbers is not specifically rejected, so no more points for IIC3y (pert. vs extran.) awarded here.

S: And then I thought that it would be easier to get these numbers (POINTS TO THE GREEN LOOP), because these [green] are higher.

R: Uh huh.

S: And then um the one [inside the orange loop], well, I saw the one right away, that you couldn't get it.

R: Uh huh.

S: And then I started multiplying and I checked 'em, and the 16 and the 17. And then that's when I called you.

R: Uh huh. Okay! And um, okay, and how did you know to start with that? What were you thinking of?

IIC3y (pert. vs extraneous) 2 pts. Another game feature is extraneous. Also confirms IIB3 (gather info).

S: Well, first I checked this (ROTATES THE POINTER ON ONE SPINNER) if there was -- if it was right and for both players and then -- nothing wrong with this (INDICATES PLAYERS) or this (INDICATES SPINNERS) -- and the coin. So, then something had to be wrong with this, uh, the numbers.

R: Uh huh. Okay. Now, one last question. One person who worked on this game thought at first that the problem had something to do with how much the coin weighs. And then, then he thought about it and he realized that the weight wasn't important. So, you see, he had an idea but then he realized that the idea wasn't helpful. Did anything like that happen to you while you were working on this?

S: No.

R: No? Like, did you try anything, or think about anything that you later realized wasn't helpful?

S: Just that um the numbers, how many on the -- in the loops.

R: Uh huh. Okay, so then, if I ask you then, okay, [name of S], what is wrong with the game, what is wrong with the game?

S: The -- some of the numbers can't be made up on the spinners.

R: Uh huh. Okay. Meaning that, when you do this stuff (INDICATING SPINNERS AND COIN), you can't get some of those numbers? (POINTS TO BOARD.)

S: Uh huh.

R: Is that right? Okay. Well, now, I'd like you to try something else. You've told me what you think is wrong with the game, right? So, now, Dr. Game wants to hire you to fix the game, okay? Again, I'm going to go over there while you're working on this. And if you need anything, or want to ask me any questions, go right ahead, and take as much time as you like. There's no need to rush. And when you think you've come up with a way to fix the game, let me know, and I'll come back, and we'll discuss what you've done, okay? Again, there's stuff available over here (POINTS TO EQUIPMENT KIT). If you need to use it, you may help yourself, if you want to, that's fine. If not, that's okay, too. All right?

S: Okay.

S WORKS INDEPENDENTLY FOR 7:20 MINUTES:

GETS PAPER AND PENCIL;

IIC1x (Diagram, etc.) 2 pts. Draws picture.

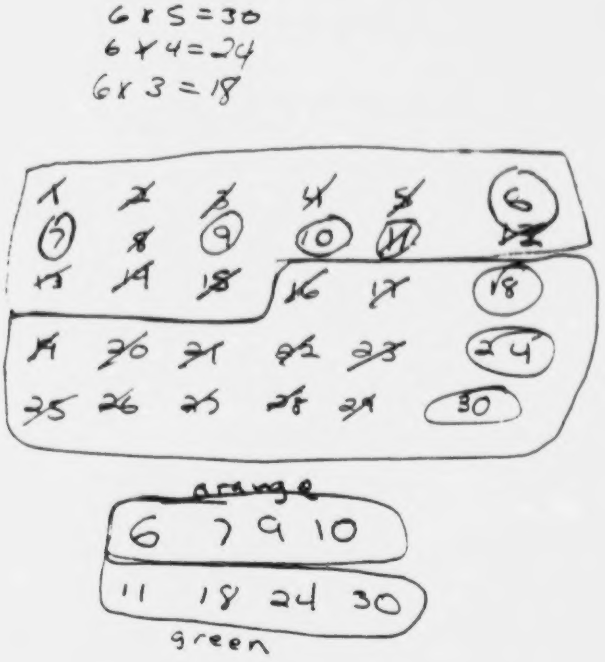
DRAWS PICTURE OF THE NUMBER BOARD, WITH LOOPS.

No points for IIB4x (calculation) awarded for this; not clear what is being calculated.

USES CALCULATOR BRIEFLY.

CIRCLES SOME OF THE NUMBERS IN HIS PICTURE.
WRITES THREE CALCULATIONS AT THE TOP OF THE PAPER.
CROSSES OFF SOME OF THE NUMBERS.
WRITES TWO ROWS OF NUMBERS UNDER THE PICTURE.
LOOKS BACK AND FORTH BETWEEN HIS PICTURE AND THE NUMBER BOARD.

HIS PAPER NOW LOOKS LIKE THIS:



S: I figured it out.

R: So, did you fix the game?

IIB4y (Manipulates) 2 pts. Makes changes to the game.

S: Okay, you can limit 'em down to four numbers and then like the game would be more of a challenge and it'd be -- and all these numbers could be made up on the spinner.

R: Okay, okay. Can you explain to me what you did here? How would that fix it? Well, why don't you explain to me what you did, first?

2 pts for IIC1x (diagram, etc.) have already been given.

S: Well, I made -- I made this board (POINTS TO THE REAL BOARD) the same way here (POINTS TO HIS PICTURE).

R: Right.

S: And then I eliminated the ones that were no good, and I circled the ones that were good, that could be made up on here (POINTS TO THE SPINNERS).

R: Uh huh.

IIB4x (calculation) 2 pts. Counting in connection with probabilities.

S: And this one (POINTING TO THE ORANGE LOOP IN HIS PICTURE) came out with five [numbers] that were good, and this one (POINTING TO GREEN LOOP) came up with three.

R: Uh huh.

IIB5 (Considers probabilities) 2 pts. Use of probability invoked explicitly.

S: And that would have been a better chance for the orange team to win.

R: Okay.

IIB6 (Guess and check) 2 pts. Adjusts the outcomes and checks.

S: So, you could put one of the orange ones [numbers] on the green and it would be four and four of the numbers...

R: Uh huh.

S: ...that can be made up on the spinners.

R: Okay. So -- I see, so you put six, seven, nine, ten (POINTING TO THE CIRCLED NUMBERS).

S: And I put the eleven down to the green team.

R: I see. I see, so that's how it would fix the game.

S: Hm hm.

R: Okay! And you're getting rid of all of these (INDICATING THE CROSSED-OFF NUMBERS)?

S: Uh huh.

R: Okay. All right. Okay! Well, I was over there [across the room], and I want to know what you were doing and what were you thinking and how you got to here. So what did you do first? And what were you thinking?

S: Well, when you told me that -- to fix the game -- I just thought right away that to take out all the numbers that are no good, and use the numbers are good.

R: Hm hm.

Points for this type of calculation awarded earlier.

S: And I did that like -- I did it by multiplying with the calculator and...

R: Hm hm.

S: ...adding. And the ones that were good, we just leave 'em in the game.

R: Hm hm. And you said by multiplying and adding, what were you doing? Multiplying and adding?

S: The -- these. (POINTS TO SPINNERS)

R: Uh huh. Okay.

S: And after -- after [having done that], I fixed these, the orange team (POINTS TO ORANGE LOOP).

R: Hm hm.

IIC2 (Simplifies) 2 pts. Simplifies the problem by considering extreme case.

S: I didn't add no more because on these two [spinners] the highest number you could add it is eleven, and eleven is there [in the orange loop].

R: Hm hm.

S: So, I just multiplied like that. Right here, and then if any of those numbers were here, then I'd circle those.

R: Uh huh. Uh huh. Okay. Okay. So, then, okay! Um, so did you do anything after that? What were you thinking?

IID4 (Related problem) 2 pts. Considers related problem.

S: No, I didn't did anything after that because I just -- fixed it and then I was thinking of putting more numbers and making another coin of subtraction and division but then it'd be -- the game would end quicker and it'll be easier with like this, with fifteen numbers.

R: Uh huh.

S: It'll be easier and -- the game would end faster. It wouldn't be like a challenge.

R: Uh huh. So, this would be more of a challenge you think. But you were going to put...?

S: Subtraction and division.

R: And what would that do?

S: It would -- like I could -- if there was subtraction and division, some of these numbers could stay in.

R: Oh, I see. Some of the numbers you eliminated here?

S: Uh huh.

R: Uh huh. Okay. Okay! Well, I have one last question. Did you think about anything while you were working this that you later realized wasn't helpful? Did anything like that happen while you were doing this?

S: No. Uhn uhn.

R: No? Okay. Okay! Well, let me get this [the equipment] out of the way...

[Interview continues with PSA B'.]

* * *

The P-score for this interview is 24 (12 actions or heuristics, each earning 2 points).

The M-score is 15; it is derived from the system of M-scores appearing in the M-Score Coding Guide, Appendix II.H. Specifically, the following subscores were obtained (the letters correspond to those in the Coding Guide):

> E (4 points) for a statement explaining why Orange is generally favored;
>
> H (2 pts) for a more complete statement about how the game favors Orange;

Q (4 points) for a set of statements that show (assuming equally likely outcomes and based on the numbers that S thinks are obtainable) that the probability of Orange winning a chip is more than 1/2;

D' (1 point) for a change in the arrangement of numbers in the green and orange loops to make the distribution of obtainable numbers more evenly split between the two loops;

J' (2 points) for a partial account of why this change makes the game fairer; and

N' (2 points) for a verification of this.

CHAPTER 5

RESULTS

Introduction

As the reader will recall, the data obtained in this study are the product of well over one hundred hours of interviews with the 48 children in the viewing and nonviewing groups. We have attempted to summarize and simplify the patterns evident in the data wherever possible. Interested readers can find details on the actual statistical analyses used here (and throughout this chapter) in Appendix II.K.

In general, our data indicate that while viewers and nonviewers performed similarly in the pretest, this picture changed markedly after viewers were exposed to 30 programs of **SQUARE ONE TV**. Viewers demonstrated statistically significant gains between the pretest and posttest, both in their use of problem-solving behaviors and in the mathematical completeness and sophistication of their solutions. By the posttest, viewers' overall performance on the PSAs was significantly higher than nonviewers'.

This chapter presents a detailed look at the data on children's performance on the PSAs. After presenting several prefatory findings, we will consider data concerning both the children's use of problem-solving actions and heuristics (i.e., their P-scores) and the mathematical completeness and sophistication of their solutions (i.e., their M-scores). Finally, we will present analyses aimed at providing further insight into the children's use of the various problem-solving actions and heuristics. The general implications of the data will be discussed to some extent here and at greater length in Chapter 6 (Discussion and Implications).

Prefatory Findings

Before proceeding to the main results of the study, two questions naturally arise and should be answered:

- Are the primes different from the non-primes?

  As discussed in Chapter 2 (The Pilot-Test Phase), we created two versions of each PSA (primes and non-primes); the two versions were designed to be dissimilar on their surface but based upon similar mathematical principles. Furthermore, they were designed to be of equal difficulty, and an analysis of data from the pretest confirmed that this was the case. Statistical tests showed there to be no difference between children's pretest performance within each of the three pairs of PSAs; that is, children did not perform differently on PSAs A and A', for example. For this reason, data were collapsed across analogous pairs (e.g., PSA A and PSA A' are reported here as PSA A*).

- How much time did children actually spend on the PSAs?

  On average, children spent eight minutes, 37 seconds working on each PSA: 7:14 on PSA A*, 6:21 on PSA B*, and 12:19 on PSA C*. (These times do not include discussions with the interviewer.) The amount of time spent did not differ significantly between viewers and nonviewers.

Problem-Solving Actions and Heuristics (P-Scores)

In Chapter 4 (The Coding Systems), we explained that children's use of problem-solving actions and heuristics was measured via three scores: (a) a Row-score reflecting the sheer number of Goal II problem-solving behaviors used, (b) a Column-score reflecting the variety of Goal II behaviors used, and (c) a composite Problem Solving-score (or P-score) which incorporates both the number and variety of actions and heuristics used.

As one might expect from their definitions, the three scores turned out to be highly related to each other ($p < .001$)[18]. Indeed, they were so highly related to each other that analyzing all three types of scores would have been misleading. Of the three types of scores, we chose to analyze children's P-scores, because these scores include the largest amount of information about children's performance.

Recall that children's behaviors on the three PSAs were coded using the same set of Goal II actions and heuristics. Nonetheless, the PSAs present problems that are quite different from each other, both in their surface features and in their underlying mathematical content. PSA A* is a combinatorics problem involving circus performers or stripes on a shirt (our least complex PSA), PSA B* involves sorting party guests or price tags into piles which meet several conditions (a problem of medium complexity), and PSA C* asks children to figure out what is wrong with a complex mathematical game and to fix it (our most complex PSA). We will consider children's performance on each PSA individually and then more generally across the three.

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[18] We have set our α level for significance at .05; any results reported here as significant achieved at least that level of significance. Results with p values between .05 and .10 are referred to as marginal.

Effects of **SQUARE ONE TV**

Table 5.1 shows, for each PSA, the percentage of viewers and nonviewers who received higher P-scores in the posttest than in the pretest. That is, the table shows the percentages of children who used a greater number and variety of problem-solving behaviors in the posttest than they had previously:

Table 5.1

Percentage of viewers (N = 24) and nonviewers (N = 24) receiving higher P-scores in the posttest

	Viewers	Nonviewers
PSA A*	79%	21%
PSA B*	50%	13%
PSA C*	71%	38%
Average	**67%**	**24%**

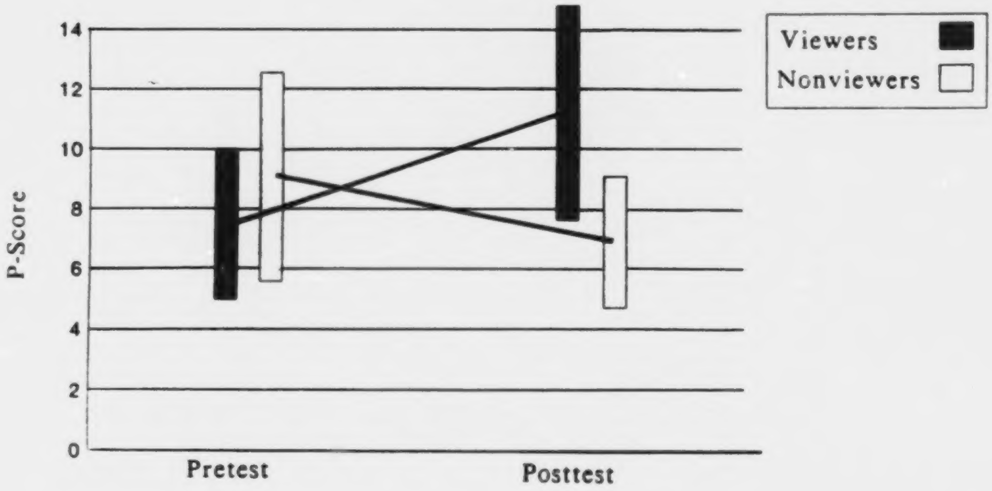
As this table illustrates, some children in each group showed a pretest-posttest increase in the number and variety of problem-solving behaviors they used. However, the percentage of children showing such an increase was significantly higher among viewers than it was among nonviewers ($p < .001$ for PSA A*, $p < .01$ for PSA B*, and $p < .05$ for PSA C*). Indeed, as the data will show, the small percentage of nonviewers whose performance increased were far outweighed by the much greater percentage of nonviewers whose performance either remained unchanged or declined between pretest and posttest.

We will now consider the average P-scores obtained by viewers and nonviewers in the pretest (i.e., before viewers saw **SQUARE ONE TV**) and posttest (i.e., after viewers saw the 30 programs from the series). We will consider the data from each PSA individually, although, as we shall see, the same trends are evident for each PSA.

PSA A*. Figure 5.1 shows the average P-scores obtained by viewers and nonviewers in the pretest and posttest, with one standard deviation above and below the mean.

Figure 5.1

Viewers' and nonviewers' mean P-scores
± one standard deviation:
PSA A*

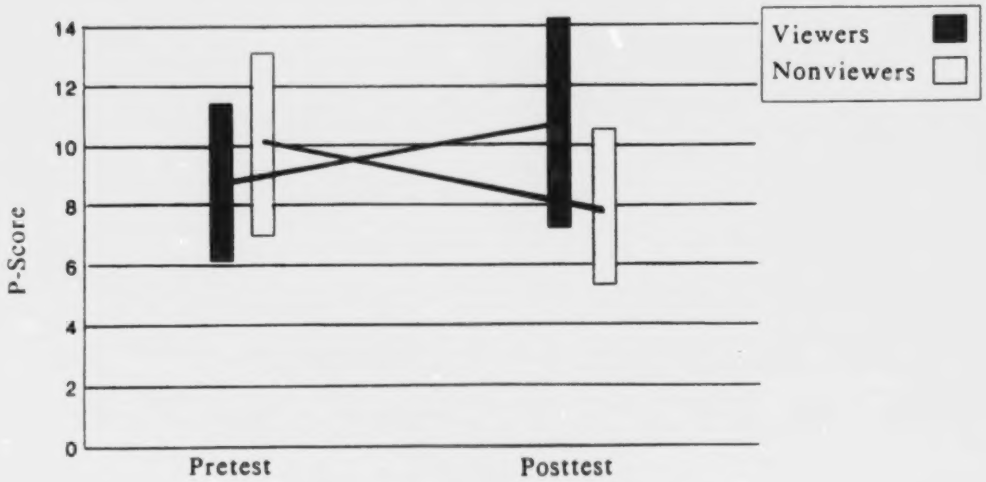


As this figure illustrates, nonviewers actually performed slightly better than viewers did in the pretest ($p < .05$). However, viewers' performance increased significantly after their exposure to 30 programs of **SQUARE ONE TV** ($p < .001$), and in the posttest, they used a significantly greater number and variety of problem-solving actions and heuristics than nonviewers did ($p < .001$). By contrast, nonviewers used significantly fewer behaviors in the posttest than they had previously ($p < .01$); we will discuss potential reasons for nonviewers' drop in performance in Chapter 6 (Discussion and Implications).

PSA B*. Figure 5.2 shows viewers' and nonviewers' average P-scores in the pretest and posttest on PSA B*, with one standard deviation above and below the mean.

Figure 5.2

Viewers' and nonviewers' mean P-scores
± one standard deviation:
PSA B*



The data obtained for this PSA are quite similar to those obtained for PSA A*. Nonviewers performed marginally better than viewers did in the pretest (p < .10). But viewers' performance increased significantly after their exposure to 30 programs of **SQUARE ONE TV** (p < .01), and they used a significantly greater number and variety of problem-solving actions and heuristics in the posttest than nonviewers did (p < .001). Nonviewers again used significantly fewer behaviors in the posttest than they had in the pretest (p < .01); as noted above, we will discuss potential reasons for this drop in Chapter 6.

PSA C*. Figure 5.3 shows the average P-scores which viewers and nonviewers obtained in the pretest and posttest on PSA C*, with one standard deviation above and below the mean.

Viewers and nonviewers used a similar number and variety of problem-solving behaviors in the pretest. However, as in PSAs A* and B*, viewers' performance increased

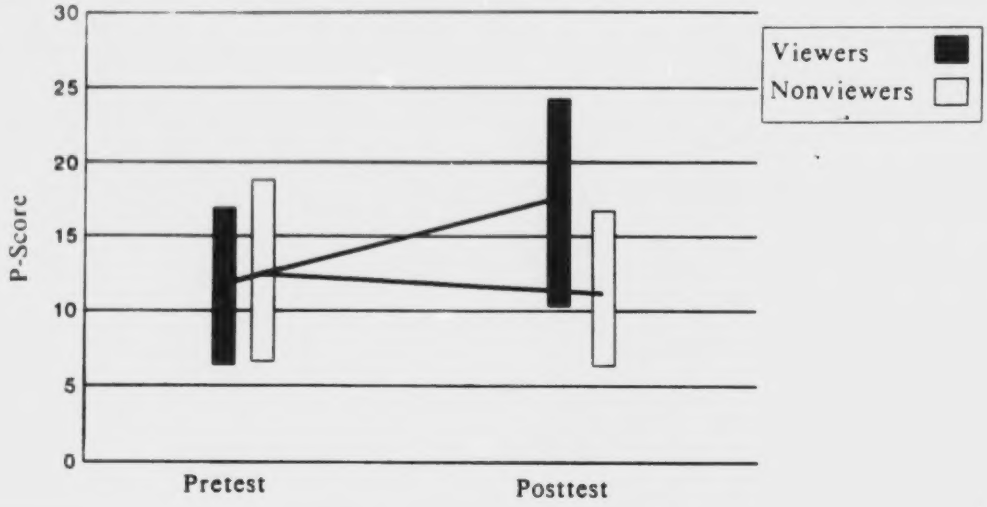


Figure 5.3

Viewers' and nonviewers' mean P-scores
± one standard deviation:
PSA C*

significantly after their exposure to 30 programs of **SQUARE ONE TV** ($p < .001$), and in the posttest, they used a significantly greater number and variety of problem-solving actions and heuristics than nonviewers did ($p < .001$). By contrast, nonviewers showed no change from pretest to posttest.

P-scores and standardized mathematics tests. Two questions were of interest with regard to children's performance on standardized mathematics tests: First, in the absence of any treatment (i.e., with regard to the pretest only), would performance on a standardized mathematics test predict performance on more sophisticated problem-solving tasks such as those presented in the PSAs? And second, would children's performance on a standardized test interact with the effects of exposure to **SQUARE ONE TV**?

To examine the first question, statistical analyses were performed to determine whether there was a significant relationship between children's scores on a standardized mathematics test[19] and the P-scores they obtained in the pretest. In fact, these tests revealed that there was not a significant relationship between their performances on the two measures.

This result may seem surprising, but we must remember that the PSAs are nonroutine problems that are quite different from those traditionally used in school. Given the largely computational nature of standardized mathematics tests, it is reasonable to expect that such tests tap different abilities from those needed for sophisticated performance in nonroutine problem-solving tasks like the PSAs. This will be discussed further in the Discussion and Implications chapter (Chapter 6).

Consistent with this finding, a second set of analyses revealed that scores on the standardized mathematics test were not significantly related to viewers' change in performance from pretest to posttest. That is, the effects of **SQUARE ONE TV** did not vary as a function of the children's performance on the standardized mathematics test.

PSAs A*, B*, and C* combined. As we have seen, each of the three PSAs produced the same effect, suggesting that viewers' exposure to **SQUARE ONE TV** resulted in their using a greater number and variety of problem-solving actions and heuristics than they would have in the absence of such exposure.

To provide a summary description of the effects seen in the three PSAs, statistical procedures were used to combine their scores on PSAs A*, B*, and C* into a single, weighted sum. These combined P-scores are presented in Figure 5.4, which can be thought of as a summary of the effects of **SQUARE ONE TV** on children's P-scores across the three PSAs. The figure illustrates the children's average combined P-scores at pretest and posttest, with one standard deviation above and below the mean.

Not surprisingly, this figure demonstrates the same pattern of results observed in each of the three PSAs individually: Nonviewers received marginally higher scores in the pretest

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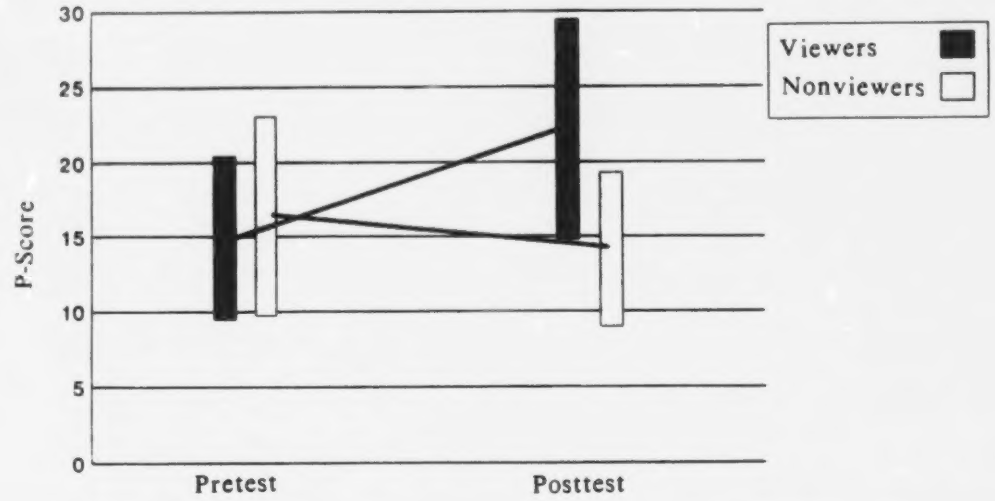
[19] The standardized mathematics test referred to here is the mathematics portion of the California Achievement Test. It was administered by the school district as part of their regular testing program before the study began.

Figure 5.4

Viewers' and nonviewers' mean P-scores
± one standard deviation:
PSAs A*, B*, and C* combined



than viewers did ($p < .10$). However, following their exposure to 30 programs of **SQUARE ONE TV**, viewers used a significantly greater number and variety of problem-solving actions and heuristics than they had previously ($p < .001$), and indeed, obtained P-scores that were greater than those obtained by nonviewers at either time ($p < .001$). Thus, it appears that viewers used these actions and heuristics as a result of their exposure to **SQUARE ONE TV**. (Nonviewers, on the other hand, showed a marginal drop from pretest to posttest ($p < .10$).)

Effects of Sex, Ethnicity, and Socio-economic Status (SES)

Having found that watching **SQUARE ONE TV** exerted a significant effect on children's use of problem-solving actions and heuristics, we next examined whether there were effects across sex, ethnicity, and SES. Analyses of the children's combined P-scores revealed that boys and girls who watched **SQUARE ONE TV** improved significantly ($p < .01$) from pretest to posttest, and there was no difference between boys and girls in the viewing group at either the pretest or the posttest.[20]

Because SES and ethnicity were largely confounded in this sample,[21] it was not surprising to find that these two factors produced similar results. Nonminority children performed better than minority children overall ($p < .05$), but this effect did not interact with the effects of watching **SQUARE ONE TV**; that is, minority and nonminority children were affected by the series in a similar way. Similarly, middle-SES children obtained higher P-scores than lower-SES children did ($p < .01$), but this effect did not interact with the effect of watching **SQUARE ONE TV**; in other words, our results suggest that **SQUARE ONE TV** affected middle- and lower-SES children similarly.

In sum, the results of our study suggest that **SQUARE ONE TV** exerted a similar effect on boys and girls, and on children of different ethnic and socioeconomic backgrounds.

Mathematical Completeness and Sophistication
(M-Scores)

In Chapter 4 (The Coding Systems), we explained that children were assigned M-scores as a function of the mathematical completeness and sophistication of their final solutions to the PSAs. One M-score was assigned for each PSA.

Effects of **SQUARE ONE TV**

Table 5.2 shows the percentages of viewers and nonviewers who received higher M-scores in the posttest than in the pretest. That is, it shows the percentages of children who arrived at more mathematically complete and sophisticated solutions in the posttest than they had previously.

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[20] There was, however, a marginally significant ($p < .10$) three-way interaction among gender, condition, and pretest/posttest; this was attributable to a drop ($p < .05$) from pretest to posttest in the nonviewing girls' P-scores.

[21] All 24 of the children in the lower SES schools were from minority groups, whereas 10 of the 24 children in the middle SES schools were from minority groups.

Table 5.2

Percentage of viewers (N = 24) and nonviewers (N = 24) receiving higher M-scores in the posttest

	Viewers	Nonviewers
PSA A*	75%	33%
PSA B*	29%	13%
PSA C*	88%	8%
Average	**64%**	**18%**

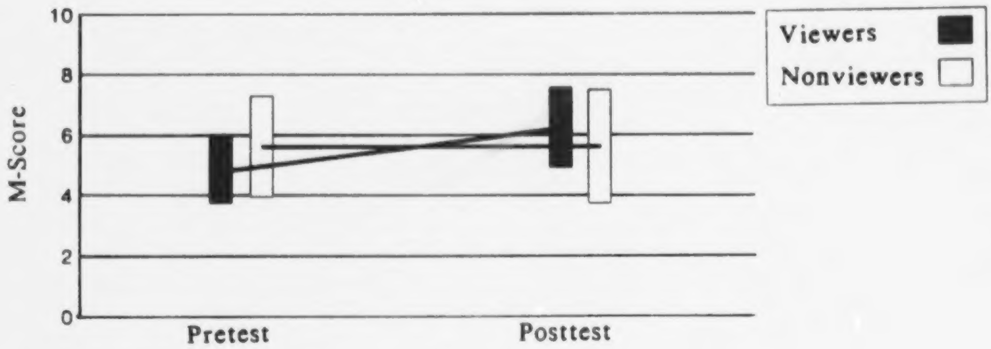
As this table illustrates, more viewers than nonviewers showed an improvement in the completeness and sophistication of their solutions. The difference between the two groups was statistically significant for two of the three PSAs ($p < .01$ for PSA A* and $p < .001$ for PSA C*); it was not significant for PSA B*, although here, too, more viewers than nonviewers showed an improvement. (We will suggest reasons for the lack of significance in PSA B* when we discuss that PSA.)

We now turn to a description of the M-scores obtained by viewers and nonviewers in the pretest and posttest. As we shall see, a similar trend is evident for PSAs A* and C*: Viewers and nonviewers did not differ in the pretest, but viewers improved significantly by the posttest, obtaining higher posttest M-scores than nonviewers did. This pattern is less evident for PSA B*.

PSA A*. In PSA A*, we presented children with four objects (circus performers or stripes on a shirt) and asked them to create as many orders as they could. One obvious way of gauging children's performance would be to compare the number of orders produced by viewers and nonviewers. But when we returned to conduct the posttest, a few children in one of the nonviewing schools told us that they had recently done an ordering problem with four objects in class, and thus already knew the answer. Possibly for this reason, we observed no effect of **SQUARE ONE TV** on the number of orders produced by viewers vs. nonviewers. However, while two groups produced similar numbers of orders, the sophistication of the techniques and reasoning used to arrive at those numbers did differ across the two groups, as reflected in the children's M-scores. Figure 5.5 shows the average M-scores obtained by viewers and nonviewers in the pretest and posttest, with one standard deviation above and below the mean.

Figure 5.5

Viewers' and nonviewers' mean M-scores
± one standard deviation:
PSA A*[1]



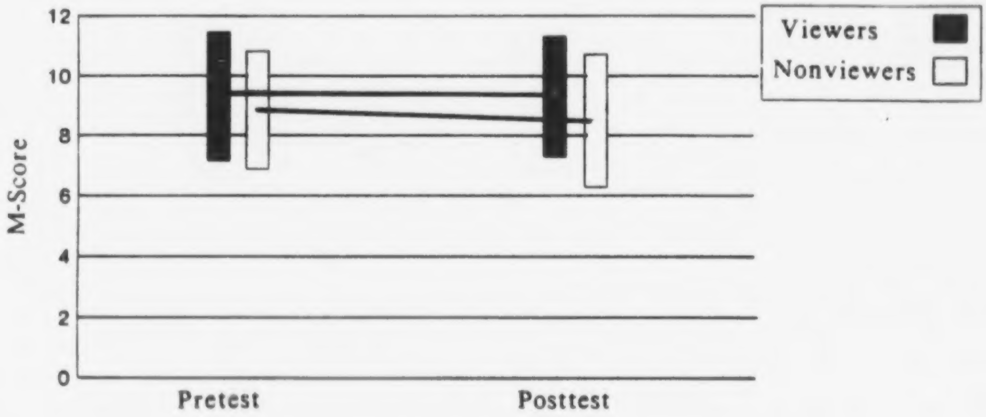
[1] Maximum M-score for PSA A* = 10.

As this figure indicates, nonviewers performed slightly better than viewers did in the pretest ($p < .05$). However, the completeness and sophistication of viewers' solutions rose significantly after their exposure to 30 programs of **SQUARE ONE TV** ($p < .001$). Their solutions were slightly more complete and sophisticated than nonviewers' were in the posttest, producing a marginal posttest difference between the two groups ($p < .10$). By contrast, the completeness and sophistication of nonviewers' solutions did not change significantly from pretest to posttest. Thus, it seems that viewing **SQUARE ONE TV** resulted in viewers' giving more complete and sophisticated solutions to PSA A*.

PSA B*. Figure 5.6 shows the average M-scores which viewers and nonviewers obtained in the pretest and posttest on PSA B*, with one standard deviation above and below the mean.

Figure 5.6

Viewers' and nonviewers' mean M-scores
± one standard deviation:
PSA B*[1]



[1] Maximum M-score without attempting optional condition = 7. Maximum M-scor with optional condition = 12.

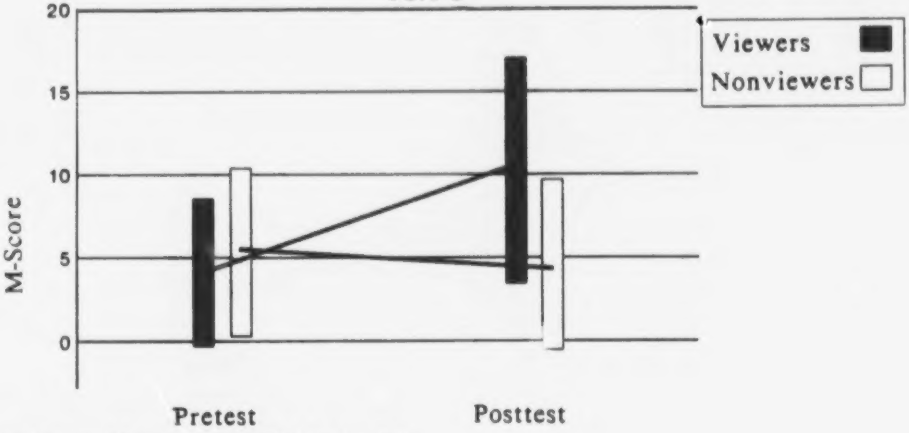
As this figure indicates, even in the pretest (i.e., before viewers were shown **SQUARE ONE TV**), both viewers and nonviewers received scores that were quite high. Children who attempted to meet the problem's optional condition (i.e., making one pile twice as large as one of the other piles) generally received scores close to the maximum of 12 points possible in this PSA. Similarly, 7 points could be obtained without attempting the optional condition, and those children who did not attempt the optional condition generally received scores near the maximum of 7. This result was unexpected since children had not scored as highly in the pilot testing that had guided our creation of the PSA.

Because children scored so highly in the pretest, there was almost no room left for improvement[22]. It is not surprising, then, that, unlike M-scores for PSA A* (and, as we shall see, PSA C*), viewers' and nonviewers' M-scores for PSA B* did not change significantly by the posttest.

PSA C*. Figure 5.7 shows the average M-scores which viewers and nonviewers obtained in the pretest and posttest on PSA C*, with one standard deviation above and below the mean.

Figure 5.7

Viewers' and nonviewers' mean M-scores
± one standard deviation:
PSA C*[1]



[1] Maximum M-score for PSA C* = 26.

As this figure indicates, viewers and nonviewers did not differ significantly in the pretest. However, after their exposure to 30 programs of **SQUARE ONE TV**, the completeness and sophistication of viewers' solutions rose significantly ($p < .001$), while nonviewers' remained relatively unchanged. As a result, viewers performed significantly better than nonviewers in the posttest ($p < .001$). Thus, it appears that viewers arrived at more complete and sophisticated solutions in this PSA as a result of their exposure to **SQUARE ONE TV**.[23]

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[22] Indeed, practically the only way in which a child's M-score could have increased would have been if he or she decided not to try the optional condition in the pretest, and later decided to attempt it in the posttest. This did not frequently occur, however; in general, children either tried the optional condition in both the pretest and posttest, or they did not try it in either test.

M-scores and standardized mathematics tests. As in the case of the P-scores, we were interested in two issues regarding performance on standardized mathematics tests: (a) whether, in the absence of any treatment (i.e., with regard to the pretest only), performance on a standardized mathematics test would be related to performance on more sophisticated problem-solving tasks such as those presented in the PSAs, and (b) whether children's performance on a standardized test would interact with the effects of exposure to **SQUARE ONE TV**.

Statistical analyses revealed that the children's scores on the standardized mathematics test were not significantly correlated with the M-scores they obtained in the pretest nor with viewers' change from pretest to posttest. Thus, in this sample, performance on the standardized mathematics test did not predict the sophistication and completeness of children's solutions to the PSAs (in the absence of any treatment), nor did it interact with the effects of viewing **SQUARE ONE TV**.

PSAs A*, B*, and C* combined. For the sake of summary, Figure 5.8 presents the children's M-scores summed across the three PSAs. As this figure indicates, viewers and nonviewers performed similarly in the pretest. However, overall, viewers' scores rose above nonviewers' following their exposure to 30 programs of **SQUARE ONE TV**. Nonviewers' posttest performance was essentially the same as their performance in the pretest.

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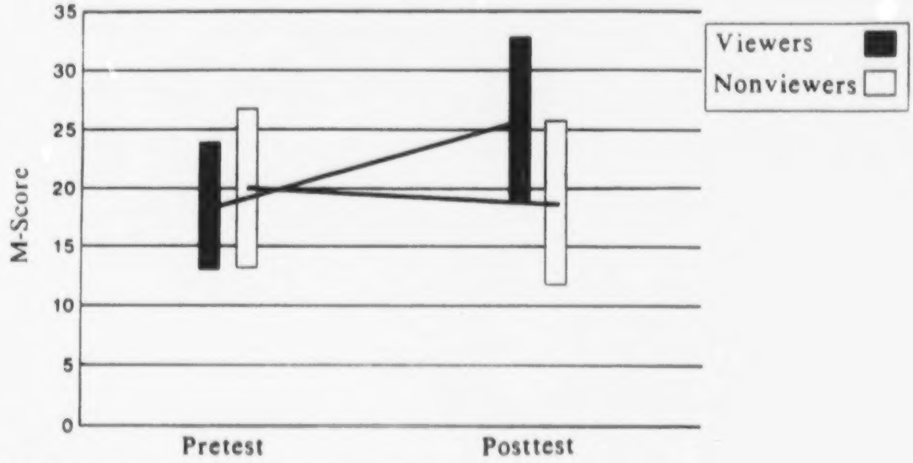
[23] The reader may wonder why the mean M-scores for PSA C* are so low, even for viewers in the posttest. The reason is that the problem is difficult for this age group, and the criteria for a full solution to PSA C* are very stringent. One part that no child mentioned, for example, is that the favored player is more likely to win the whole game (that is, to get five chips before the unfavored player does) because the former is more likely to win each chip individually. See the M-score coding guide in Appendix II.H.

Figure 5.8

Viewers' and nonviewers' mean M-scores:
PSAs A*, B*, and C* combined[1]



[1] Maximum combined M-score = 48.

However, we must be cautious in drawing conclusions about combined M-scores. Combined M-scores are more difficult to interpret than are the combined P-scores that we discussed in the Problem-Solving Actions and Heuristics section. The reason for this difficulty stems from the different natures of the P- and M-scores: P-scores reflect the degree to which children use the problem-solving actions and heuristics described in Goal II of **SQUARE ONE TV**; the same types of behavior that give rise to a child's P-score in PSA A* can also lead to his or her P-scores in PSA B* and PSA C*. In other words, the P-score can be used to quantify performance on any problem-solving task that lends itself to some combination of the problem-solving actions and heuristics that comprise this score.

In contrast, an M-score represents the completeness and sophistication of a child's solution to a particular problem; thus, this score is necessarily unique to and bound to that problem[24]. Because of the conceptual differences among the three M-scores, and because statistical tests indicated that (unlike P-scores) M-scores on the three PSAs were not all significantly correlated with each other, we must be careful about interpreting combined M-scores as indicators of some sort of "general mathematical completeness and sophistication."

Effects of Sex, Ethnicity, and Socio-economic Status (SES)

As with P-scores, statistical analyses were performed to determine whether **SQUARE ONE TV**'s effect on M-scores differed as a function of children's sex, ethnicity, or SES. Sex was found to have no effect in either the pretest or the posttest, nor did it interact with the effects of **SQUARE ONE TV**, suggesting that exposure to **SQUARE ONE TV** affects boys and girls to a similar extent.

As in our analysis of the children's P-scores, ethnicity and SES were found to exert similar effects on children's M-scores. On the whole, middle-SES children arrived at more complete and sophisticated solutions than did lower-SES children in PSAs A* and C* ($p < .01$ and $p < .05$, respectively), but this effect did not interact with the effect of watching **SQUARE ONE TV**. Similarly, nonminority children performed marginally better than minority children in PSA C* ($p < .10$), but this effect did not interact with the effects of watching **SQUARE ONE TV**. Thus, the effects of the series did not differ for children of various ethnic and socioeconomic backgrounds.

Relation Between P- and M-Scores

As discussed in the previous section, viewers used a greater number and variety of problem-solving behaviors in the posttest than they had in the pretest; nonviewers, on the other hand, did not show this increase. The M-score data presented in this section show a similar effect: With the exception of PSA B* (in which children had received high scores even in the pretest), the completeness and sophistication of viewers' solutions rose from pretest to posttest, but no improvement was found for nonviewers. Statistical tests were performed to determine whether children's P-scores and M-scores were related to each other, i.e., whether using a larger number and variety of problem-solving behaviors would be associated with children's producing more complete and sophisticated solutions. In fact, there was a significant correlation between the two ($p < .001$).

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[24] Interested readers can get a more comprehensive idea of the differences among the derivations of the three M-scores by comparing the scoring systems used for the three PSAs. These scoring systems are presented in Appendix II.H.

Further Analysis of Problem-Solving Actions and Heuristics

In this section, we will look a little more deeply into the effects reported in the previous two sections, by focusing on the nature of the effects of **SQUARE ONE TV** and the types of problem-solving actions and heuristics that seemed to increase most as a result of the viewers' exposure to the series.

New Behaviors

As we saw in the section on Problem-Solving Actions and Heuristics, viewers showed a greater gain than nonviewers in the number and variety of problem-solving behaviors they used in each of the three PSAs. However, simply knowing that a child's P-score changed from pretest to posttest is not enough to know how much of his or her posttest behavior consisted of new problem-solving behaviors (i.e., behaviors that he or she did not exhibit in the pretest)[25].

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[25] To understand this point better, consider a child who received a P-score of 6 in the pretest and 10 in the posttest. It is possible that 6 (or even more) of the 10 posttest points stemmed from the child's using the same behaviors that he or she had used in the pretest. But it is also possible that none of the pretest behaviors were repeated and all 10 posttest points reflected new behaviors.

We must look further to determine how many new actions and heuristics were exhibited in the posttest.

To investigate this point, we examined each child's pretest and posttest performance to determine the degree to which he or she introduced new behaviors in the posttest. For each child we computed the percentage of behaviors that he or she used in the posttest but not in the pretest. Table 5.3 shows the mean percentages of problem-solving actions and heuristics that viewers and nonviewers used for the first time in the posttest.

Table 5.3

Percentages of actions and heuristics used for the first time in the posttest

	Viewers	Nonviewers
PSA A*	43%	22%
PSA B*	33%	23%
PSA C*	49%	31%
Average	**42%**	**25%**

As this table illustrates, in each PSA, both viewers and nonviewers introduced some percentage of new problem-solving behaviors in the posttest. However, this percentage was greater for viewers than for nonviewers; statistical tests revealed the difference between viewers and nonviewers to be significant for PSAs A* and C* ($p < .01$) and marginal for PSA B* ($p < .10$).

Overall, viewers were more likely than nonviewers to introduce new problem-solving actions and heuristics in the posttest. The data suggest that viewers' exposure to 30 programs of **SQUARE ONE TV** resulted in their applying new actions and heuristics to these problems -- actions and heuristics that they might not have used otherwise.

Change in Problem-Solving Actions and Heuristics Over Time

As we have seen in the Problem-Solving Actions and Heuristics section, viewers showed a greater P-score gain than nonviewers did. In this section, we will look more closely at which actions and heuristics were most responsible for this gain.

Appendix II.J presents graphs showing the number of children who used each of the behaviors described under Goal II in the pretest or posttest. As those graphs make clear, viewers and nonviewers were highly similar in their choice of problem-solving behaviors in the pretest. However, after the viewers were exposed to 30 programs of **SQUARE ONE TV**, their choice of behaviors became quite different from nonviewers'. These changes are summarized in Figure 5.9; the graphs represent the increases seen for each of the problem-solving actions and heuristics described under Goal II[26].

Averaged across the three PSAs, the number of viewers using a particular behavior increased for 11.7 of the 17 problem-solving actions and heuristics coded. By contrast, nonviewers showed this change for an average of only 4.0 of the 17.

The degree of change observed for each individual action or heuristic varied across the PSAs. This is not surprising, for several reasons. First, the three PSAs are very different problems; each lends itself to different behaviors, so we would not expect to find children using exactly the same behaviors on all three PSAs. Second, the children may be initially inclined toward using some actions or heuristics over others (e.g., an individual child might be more likely to draw a diagram than to rely on computation). Third, viewers' use of the behaviors may relate to the extent to which each was shown in the programs of **SQUARE ONE TV** used in this study. We will consider each of these factors in greater detail in Chapter 6 (Discussion and Implications).

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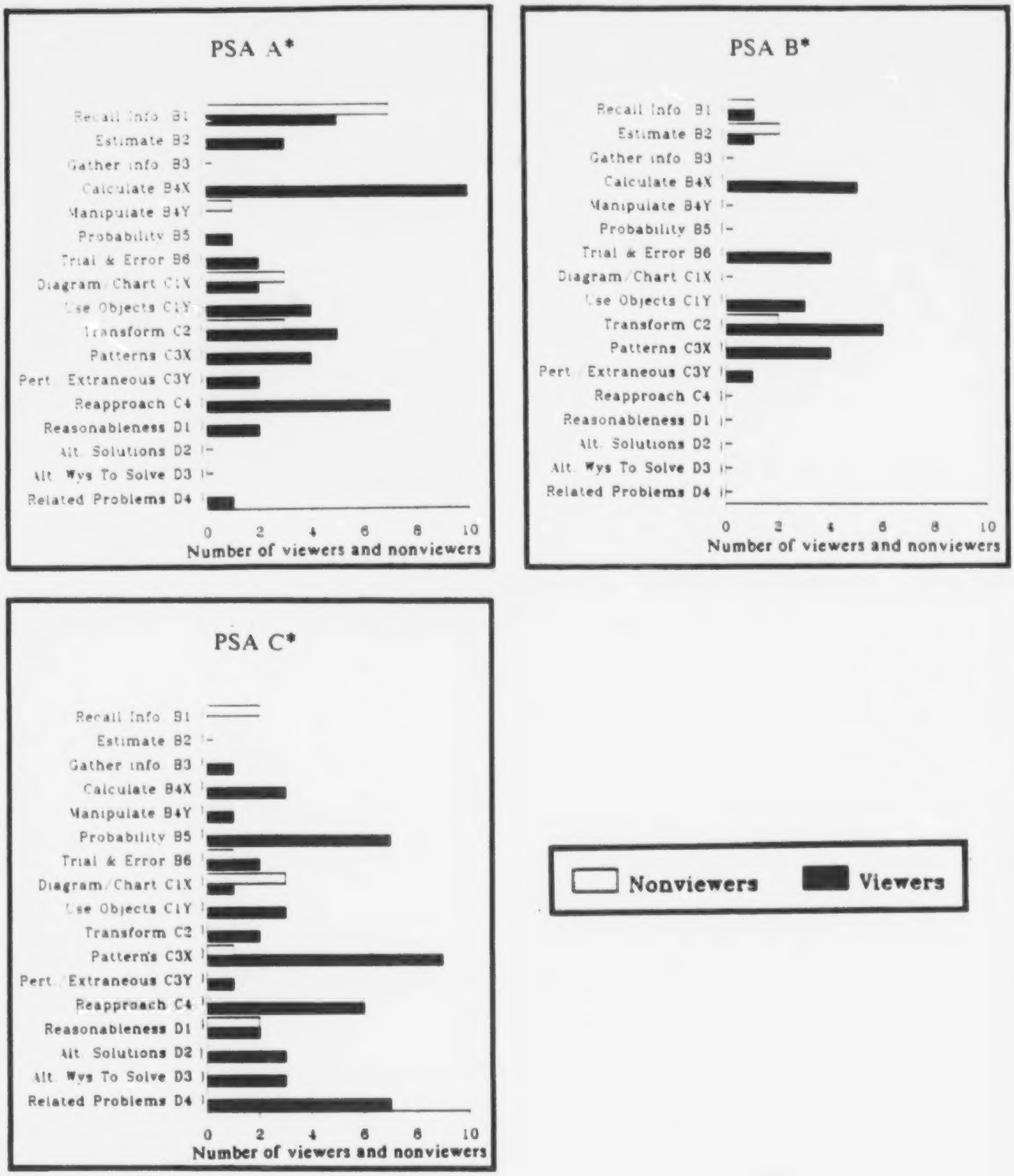
[26] Note that because these graphs only show pretest-posttest increases in the number of children using a particular behavior, "0" is used to indicate either no change or a decrease in the use of that behavior. These can be distinguished via the graphs shown in Appendix II.J, however.

Figure 5.9

Increase in the number of viewers and nonviewers using particular problem-solving actions and heuristics, for each of three PSAs



CHAPTER 6

DISCUSSION AND IMPLICATIONS

"What did one arithmetic book say to the other?"

Answer: "I have a lot of problems."

-- Laffy Taffy,
Beich's Candy Co.

Introduction

The plan of this final chapter is as follows: First, we summarize very briefly the results presented in the previous chapter. Next, we discuss the relationship between problem-solving scores and mathematical scores (i.e., between P-scores and M-scores). Then we propose a variety of ways to account for these results, attempting to explain the changes that were observed in viewers' and nonviewers' performance from pretest to posttest. As part of this section, we look at how **SQUARE ONE TV**'s specific Goal II subgoal content relates to specific changes in viewers' use of problem-solving actions and heuristics. We follow with a discussion of the level of viewers' exposure to **SQUARE ONE TV** and some remarks on implications for further research. We conclude with a summary discussion of the relations among school mathematics as currently practiced, the PSAs, and **SQUARE ONE TV**, all in the context of the broader reform movement in mathematics education.

Summary of Results

The following is a brief summary of some of the results contained in the previous chapter. (Recall that the P-score measures the number and variety of problem-solving actions and heuristics used, and the M-score is a measure of the mathematical sophistication and completeness of the solution reached.) The full details on these results appear in the previous chapter and in Appendix II.K.

P-score gains. From pretest to posttest, children in the viewing group made significantly greater P-score gains on each of the three PSAs than the nonviewers did. Further, the viewers' pretest to posttest gains were significant for each PSA. The nonviewers did not make significant gains on any of the PSAs, and in fact their P-scores decreased significantly on PSAs A* and B* (but not C*). In each PSA, the difference between the viewers' and nonviewers' P-scores was significant at the posttest.

M-score gains. From pretest to posttest, children in the viewing group made significantly greater M-score gains than nonviewers on two of the three PSAs (namely PSAs A* and C*). On those two PSAs, the viewers' M-scores increased significantly from pretest to posttest, and the posttest difference between the two groups was significant on PSA C*. The nonviewers' M-scores did not change significantly on any of the PSAs.

P-score/M-score correlation. In this sample, P-scores and M-scores were significantly correlated, with higher P-scores associated with higher M-scores.

Effects of sex. There were no significant main effects of sex on either P-scores or on M-scores. Furthermore, both boys and girls who watched **SQUARE ONE TV** improved significantly on both measures.

Effects of SES and ethnicity. Middle-SES children received significantly higher combined P-scores than low-SES children did and significantly higher M-scores on PSAs A* and C*. However, SES did not interact significantly with the effects of **SQUARE ONE TV**. Comparisons of children by ethnicity yielded similar results; this was expected because ethnicity was so highly confounded with SES in this sample.

Correlations with standardized tests. Children's scores on the standardized mathematics test administered by the school district were not significantly correlated with P-scores or M-scores on any of the PSAs. Further, the pretest-posttest changes in viewers' P-scores and M-scores were not correlated with standardized test scores.

Relationship Between P-Scores and M-Scores

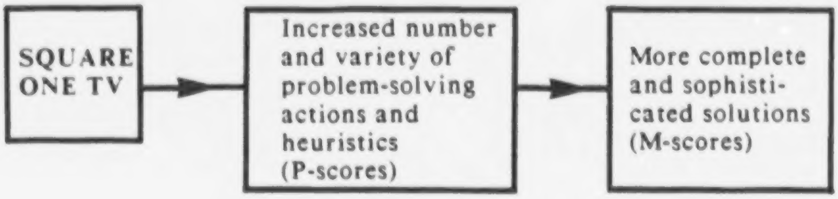
Earlier we argued that P-scores and M-scores are conceptually independent in the sense that a low or high score of one kind does not necessarily imply anything about a corresponding score on the other. One person might engage in a large number of problem-solving behaviors in the course of working on a PSA, without getting very far mathematically, while another person might arrive at a relatively complete and sophisticated solution using only a small number of problem-solving actions and heuristics. In fact, however, there is a statistically significant positive correlation between P-scores and M-scores ($p < .001$) in this sample of children. However, this does not necessarily mean that one causes the other. While it is possible that using a variety of problem-solving actions causes children to reach sophisticated solutions, it is just as possible that the causality works in the opposite direction -- that children who are capable of reaching such solutions also tend to use a greater variety of actions. Indeed, it is also possible that exposure to **SQUARE ONE TV** affected each aspect of problem solving independently.

Still, we can make some hypotheses about cause and effect between the children's P- and M-scores, because of the nature of the two measures. As is evident from the M-score coding scheme shown in Appendix II.H, a child's M-score reflects the completeness and sophistication of his or her final solution to a problem; thus, the score represents the outcome of the child's attempt to solve the problem. By contrast, a child's P-score is derived from the actions and heuristics that he or she used to reach a solution; thus, P-scores represent the process of problem solving. Because the process of problem solving must necessarily precede and contribute to its outcome, it seems reasonable to assume that a child's P-score contributes to his or her M-score. In other words, the use of a greater number and variety of problem-solving actions may help lead a child to a more complete and sophisticated solution.

If this is the case, then we can posit a model that looks like Figure 6.1, below:

Figure 6.1

How **SQUARE ONE TV** might affect children's problem solving


SQUARE ONE TV
Increased number and variety of problem-solving actions and heuristics (P-scores)
More complete and sophisticated solutions (M-scores)

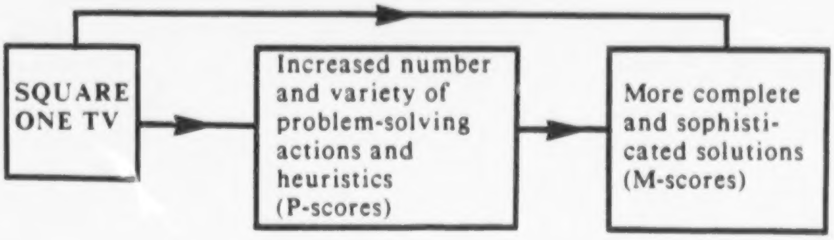
(Note that at this point all we have done is to suggest that the connection between P-score and M-score can be represented by the arrow on the right; we have not yet discussed the mechanisms through which viewing **SQUARE ONE TV** leads to an increase in P-score. This will be pursued later.)

To investigate this relationship further, we performed a statistical model fitting analysis (Bentler, 1985) for each of the PSAs, using three variables: condition (viewer or nonviewer), change in P-score from pretest to posttest, and change in M-score. The statistics indicated that the model shown in Figure 6.1 fit the data well for PSAs A* and B*, and less well for PSA C*. (See Appendix II.K for full details on this.) This analysis suggested that while the two arrows of Figure 6.1 represent significant effects of one variable upon another, the model is not complete.

For this reason, a second model was explored, one in which increases in M-score were hypothesized to result both from an increase in P-score and directly from viewing **SQUARE ONE TV**. That is, the model of Figure 6.1 was augmented by a third arrow, as in Figure 6.2.

Figure 6.2

A somewhat more refined model of how **SQUARE ONE TV** might affect children's problem solving


SQUARE ONE TV
Increased number and variety of problem-solving actions and heuristics (P-scores)
More complete and sophisticated solutions (M-scores)

The same procedure was used to test this more refined model, and here the statistics indicated that the model of Figure 6.2 fit the data from all three PSAs. The results of this analysis indicate that **SQUARE ONE TV** exerted (a) a direct effect upon the children's use of problem-solving actions and heuristics, (b) a direct effect upon the sophistication and completeness of their solutions and (c) an indirect effect upon children's M-scores, mediated by its effect on P-scores (i.e., by increasing the number and variety of problem-solving behaviors children used, exposure to the series allowed them to reach more sophisticated solutions as well).

SQUARE ONE TV's Effects on P-Scores and M-Scores

Now that we have some understanding of the interconnections among exposure to **SQUARE ONE TV** and changes in P- and M-scores, we turn next to a variety of hypotheses regarding the mechanisms by which viewing **SQUARE ONE TV** might affect children's P-scores and M-scores.

The interviewer-**SQUARE ONE TV** connection hypothesis. One hypothesis is that the viewers realized that there was some connection between the people who conducted the interviews and **SQUARE ONE TV**. (Of course the nonviewers could not make this connection.)

This would cause the viewers to be especially attentive, or to work especially diligently, in an attempt to impress or please the interviewers.

This hypothesis is very unlikely. Great pains were taken to avoid anything that would lead children to suspect any such connection, and in fact there was no indication that the viewing children made any association between the interviewers and the series. Even during the **SQUARE ONE TV** Interview (which was the last instrument, used with the viewing group only), children spoke to interviewers as though the interviewers were unfamiliar with the series. After all the interviews were completed, the viewers were told of the connection, at that point they evinced great surprise.

The interviewer/coder expectation hypothesis. Here the hypothesis is that the expectations of the interviewers or coders somehow biased their perceptions of the children's responses. In fact, though, this is not the case, for reasons that we have already explained. Throughout the data collection and coding phases, interviewers and coders were kept blind as to whether individual children were viewers or nonviewers. Thus the observed effects are not the product of experimenter bias.

The PSA-**SQUARE ONE TV** segment connection hypothesis. Another hypothesis is that there were segments appearing in the 30-program treatment that were so similar to one or more of the PSAs that the viewing children simply repeated exactly what they had been shown in those segments. (Presumably, this would have a direct effect on M-scores, as opposed to an effect mediated through an increase in problem-solving behavior.)

This hypothesis is also unlikely, however. As we have noted earlier, none of the segments in the 30 programs is isomorphic to any of the PSAs. The closest match is between PSA A* (ordering four stripes or performers) and the segment "Yes, General, Sir," which involves finding the number of ways in which three objects can be linearly ordered. If anyone noticed that connection, no one mentioned it.[27]

We turn now to some more plausible substantive hypotheses concerning the underlying reasons for **SQUARE ONE TV**'s apparent effect on viewers' P-scores and M-scores. Consider first the question of how viewing the series might have affected M-scores.

The mathematical content hypothesis. We have just noted that there are no individual segments in the 30 programs that are identical to the content of any of the PSAs. We hypothesize, then, that exposure to some group of segments has an effect on children's success with the PSAs.

Indeed, this seems reasonable. For example, to attain an M-score of more than 3 on PSA C*, one must use probabilistic ideas (see Appendix II.H); it appears that the exposure over the course of the treatment to a variety of segments involving probability may have had a direct effect on children's M-scores in this PSA. (See Appendix I.B of Volume I for a description of individual segments in the treatment; some of these involve fair random devices, independent events, and basic calculations of probabilities.)

Consider now some hypotheses regarding the effects of **SQUARE ONE TV** on children's P-scores.

The modeling of problem solving hypotheses. The 30 programs that the viewers watched during the course of the treatment included a total of 116 problem-solving segments. Among all the segments shown during the six weeks there were more than 500 distinct instances of Goal II subsubgoals from Problem Treatment, Heuristics and Problem Follow-up,[28] an average of more than 17 per day. Thus a substantial amount of problem-solving is modeled on **SQUARE ONE TV**.

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[27] In this respect, this study is different from the Comprehension and Problem-Solving Study (Peel, et al., 1987). As part of that study, children were asked to extend the mathematical ideas contained in an isolated segment that had just been shown to them.

[28] This count does not split IIB4 (calculation or manipulation) into IIB4x (calculation) and IIB4y (manipulation), nor does it include 166 instances of subsubgoals from Problem Formulation (IIA).

This continued modeling of problem-solving behavior might have had either or both of the following effects on the audience: (a) The children might have added to their own problem-solving repertoires behaviors that were not present at the pretest; or (b) children might have become more inclined to use problem-solving behaviors that were already in their repertoires, but which for some reason they did not consider applicable in the pretest. That is, a child might see characters using the heuristic of working backwards, for instance. As a consequence, he or she might either (a) add working backwards to his or her own problem-solving toolbox, or else (b) realize that the technique might be useful and appropriate in some problem-solving situation(s) to which he or she had not previously considered it applicable. Presumably these processes might be conscious or unconscious.

To distinguish between these two alternatives would require (at the very least) a much more detailed analysis of content of the six weeks of segments, and it seems unlikely that such a distinction could be made reliably. Actually, there appears to be no compelling reason to distinguish between these two hypotheses. Under either one, **SQUARE ONE TV**'s impact on children's problem solving is to increase the number and variety of problem-solving actions and heuristics they use. Certainly both hypotheses are compatible with the data.

The motivation hypothesis. A further variant on the last two hypotheses suggested immediately above is that viewers do in fact have certain problem-solving actions and heuristics in their repertoires, and see their applicability to the PSAs (both at pretest and posttest), but are not sufficiently motivated to use them initially. Under this hypothesis, an effect of **SQUARE ONE TV** is to increase children's motivation to use those problem-solving behaviors when working on the PSAs.

This is part of a more general set of hypotheses concerning the attitudinal effects of **SQUARE ONE TV**. The essential idea here is that the problem-solving in **SQUARE ONE TV** is consistently embedded in a humorous, lively format that we know is highly attractive to children in the target age group. Further, characters regularly use mathematics successfully and take evident pride in their mathematical accomplishments. Through exposure to this appealing context, viewers' attitudes toward mathematical problem solving are somehow altered in such a way that they become more inclined to use problem solving behaviors in the posttest than they were in the pretest. As in the case of the previous hypotheses, the effect may be conscious or unconscious, and of course motivation may play a different role for different children. Nonetheless, the basic hypothesis is that the cognitive effect of viewing **SQUARE ONE TV** is mediated in part by an attitudinal effect.

Supporting the hypothesis that motivation plays a role in problem solving is an examination of our magnitude ratings of motivation.[29] There was, in fact, a significant positive correlation ($p < .01$) between the children's posttest P-scores and their magnitude ratings for motivation; as motivation increased, so did the number and variety of problem-solving actions and heuristics viewers and nonviewers used.

It seems likely that it is actually some combination of these last three hypothesized mechanisms -- gaining new problem-solving behaviors, gaining knowledge of the applicability of existing problem-solving behaviors, and gaining the motivation to use existing and applicable problem-solving behaviors -- that is responsible for the changes seen in viewers' P-scores. Even within a single child there is probably a complex mixture of effects at work.

A more refined conception of the effects of **SQUARE ONE TV** on P-scores and M-scores, then, would have to allow for a role for attitudes as well. A more complete diagram might therefore look something like Figure 6.3. This figure suggests that **SQUARE ONE TV** might affect P-scores, M-scores, and attitudes directly; attitudes may in turn affect P- and M-scores. Certainly, other interactions are possible as well.[30]

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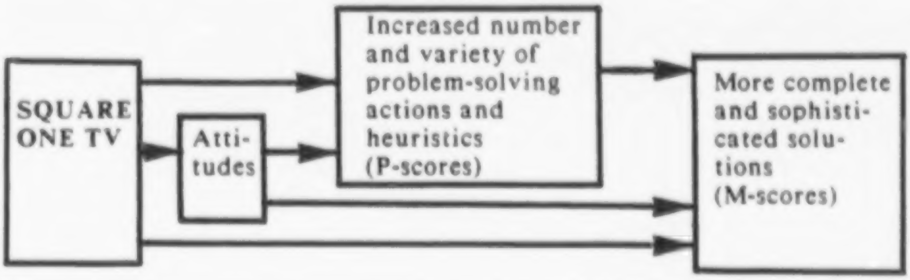
[29] Essentially, the motivation magnitude rating combines the sophistication of a child's expression of affect with its valence (positive or negative); Volume III contains a complete description of the measure.

[30] There is no statistical procedure comparable to the one used on Figures 6.1 and 6.2 that can be used to test this more complex model; hence it should be interpreted as a hypothetical construct.

An obvious next step toward obtaining a more detailed picture of the effects of **SQUARE ONE TV** would be to try to determine which actions and heuristics were most affected by the children's exposure to the series. We address that issue next.

Figure 6.3

A Further Refinement of a Model of How **SQUARE ONE TV** Might Affect Children's Problem Solving


SQUARE ONE TV
Atti-tudes
Increased number and variety of problem-solving actions and heuristics (P-scores)
More complete and sophisticated solutions (M-scores)

Which Actions and Heuristics Are Most Affected?

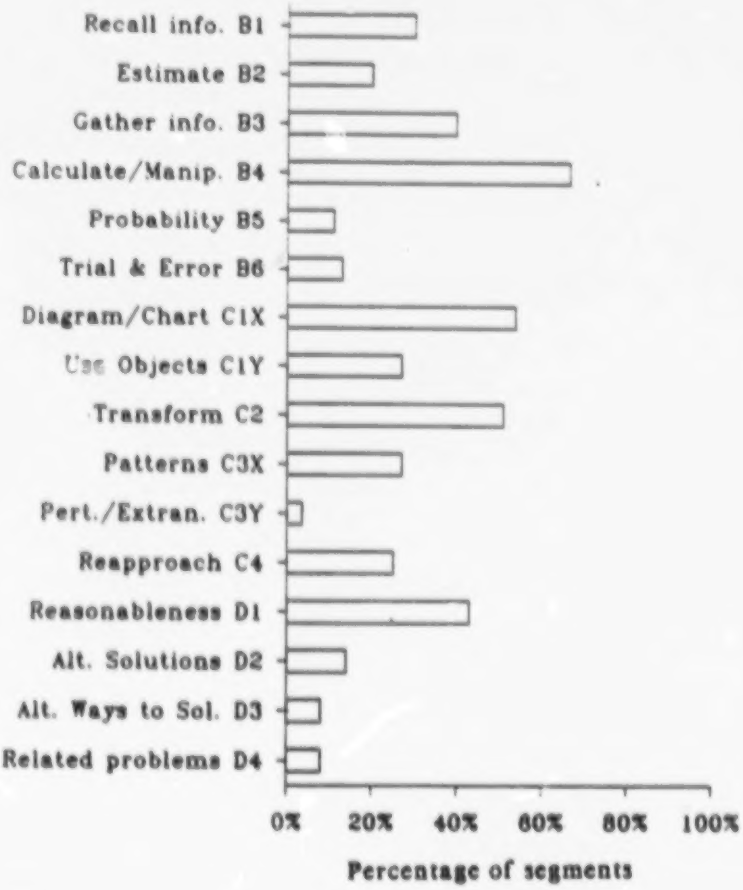
As readers may recall, the bar graphs in Appendix II.J show how many viewers and nonviewers used each of the 17 Goal II subgoals in each of the PSAs, for pretest and posttest. The information in those six graphs is condensed into the graphs in Figure 5.9, which show increases, from pretest to posttest, in the number of subjects who used each of the subgoals.

One of the figures in Appendix I.C (in Volume I) presents the percentage of segments in the programs shown to viewers that contain Goal II subsubgoals. The information from that figure is repeated here, as Figure 6.4.

In examining Figure 6.4, below, and Figure 5.9 of Chapter 5, it is natural to ask the question: What is the relationship between them -- that is, the relationship between (a) the extent to which the Goal II subgoals are represented in the 30 programs that the viewing group watched, and (b) the extent to which particular subgoals are used by more viewers than nonviewers?

Figure 6.4

Percentage of Segments Containing
Certain Goal II Subsubgoals



This question turns out to be a very difficult one to answer in any simple way. In the paragraphs that follow we discuss some of the important issues to bear in mind, as well as several specific examples of the kinds of relationships that exist between **SQUARE ONE TV** content and viewers' problem-solving behavior.

First, consider the relative emphases placed on each of the various Goal II subgoals, as shown above in Figure 6.4. It is essential to keep in mind that Figure 6.4 depicts the relative emphases within **SQUARE ONE TV**; it does not make comparisons with those subgoals as typically represented in a standard elementary school mathematics curriculum. As the figure illustrates, there are many more **SQUARE ONE TV** segments in which characters solve problems by calculation and manipulation (IIB4), for example, than there are in which characters use probability (IIB5). Nonetheless, the amount of using probability (IIB5) in **SQUARE ONE TV** is enormous when compared with the amount in the usual curriculum, which is, in practice, virtually zero. Hence the use of probability that occurs in **SQUARE ONE TV**, though small relative to the amount of calculation and manipulation, may have an effect that is disproportionally large.

This suggests, more generally, that one would have to undertake a detailed analysis of the particular curriculum that a child is using before one could make any a priori judgments about the potential effects of **SQUARE ONE TV**. But if one were to construct a graph showing the relative emphases of the 17 Goal II subgoals in most of the standard curricula, there would be only two major peaks -- one for recall information (IIB1) and another for calculation and manipulation (IIB4). It turns out, then, that such an analysis is not needed in most cases, simply because most standard curricula are much alike in their overwhelming emphasis on the development of computational algorithms (an assessment that is amply supported by the Standards report (NCTM, 1989)).[31]

A second factor complicating the comparison between the figures is that the bar graph showing the Goal II content of the 30 programs of **SQUARE ONE TV** does not account for the relative strengths of the presentations of the individual subgoals. The length of the segment, for example, is not taken as a factor in computing the percentages, nor is anyone's judgment of how successful a segment is in modeling the subgoal. (Reasons for this are discussed in

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[31] More fine-grained analyses would be required for the few programs -- like Real Math (Willoughby, et al., 1985), the Comprehensive School Mathematics Program (CSMP, 1985), and Developing Mathematical Processes (Romberg, Harvey, Moser, and Montgomery, 1974) -- that are not as heavily weighted with recall and computation.

Schneider, Aucoin, Schupack, Pierce, & Esty, 1987.) Therefore, while the graph of Figure 6.4 accurately depicts the percentage of segments that contain certain subgoals, it may not show as accurately the relative strengths of the impressions that those subgoals might make on the viewer over the course of 30 programs.

Third, the relation between **SQUARE ONE TV** and PSA performance is not likely to be a simple, causal one. Rather, it appears that the viewer's problem-solving behaviors (and the Goal II subgoals under which they are categorized) can be affected by at least these three factors:

(a) an influence from watching **SQUARE ONE TV**;

(b) the demands of the task; that is, the kinds of behaviors that a particular problem necessitates or tends to elicit; and

(c) what the child would do normally, without any influence from **SQUARE ONE TV**.

These factors are interrelated, and often more than one operates simultaneously. Consider a few illustrative examples:

- Viewers' performance on PSA A* (ordering shirt stripes or circus performers) provides a good example of instances in which watching **SQUARE ONE TV** has apparently had an effect. In this PSA, viewers (but not nonviewers) increased from pretest to posttest in their use of IIB4x (calculation -- typically combinatorial multiplying) and IIC4 (reapproach problem -- typically by adopting a more systematic approach). Both of these are particularly appropriate actions or heuristics to use on this combinatorial problem.

- Next, consider an example that illustrates how the effects of **SQUARE ONE TV** might depend upon the appropriateness of a given problem-solving behavior for a particular problem. Compare the bars for using probability (IIB5) in PSA B* (distributing guests or clocks) and in PSA C* (the dice or spinner game).

In the case of PSA B*, there is no increase from pretest to posttest in either group simply because neither group uses probability on either occasion. Given the nature of the problem, that is hardly surprising.[32]

In PSA C*, on the other hand, there is considerable use of probability (IIB5) by both groups in both the pretest and the posttest. One would expect this because using probability is appropriate in that particular PSA. Further, at the posttest, one finds an even greater use of probability among the group who watched **SQUARE ONE TV** than the nonviewing group at the posttest; presumably, this can be ascribed, at least in part, to the presence of probability in the series.

- By contrast, some problems demand certain problem-solving behaviors to an extent that outweighs any potential effects of **SQUARE ONE TV**. Consider the use of manipulation (IIB4y) in PSA A* (ordering the shirt stripes or the circus performers) and PSA B* (grouping the guests and clocks). Here there is very little change in either group from pretest to posttest; in contrast to the previous example, however, the reason in both PSAs is that virtually everyone uses manipulation on both occasions. Hence, there is no room for growth in either group.

- Next is an example of a heuristic that is used to different extents in two different PSAs, even though it is appropriate in both. Compare the use of IIC1x (list, table, chart, etc.) in PSA A* (ordering stripes or performers) and PSA C* (the games). While there is some growth in both groups in PSA A*, listing the orders as they are obtained is very common among viewers and nonviewers.[33]

In PSA C*, on the other hand, making a table of possible outcomes (IIC1x) is a relatively sophisticated heuristic, and its use is much less common in both groups. (It should be noted as well that some subjects received points under IIC1x for drawing pictures that were less directly relevant to solving the problem -- for example, pictures of the foam-core players.)

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[32] It is conceivable, though unlikely, that one might use probability in PSA B*; an example is provided in the P-Score Coding Guide (Appendix II.G).

[33] In the pilot test, many fewer children listed the orders of the stripes or performers, so we thought that there might be differences between viewers and nonviewers in the main study. This did not happen.

- Finally, consider the use of looking for alternative solutions (IID3) in PSA B* (grouping the guests or clocks). The problem was deliberately constructed to have many solutions, thus providing an opportunity for children to engage in the problem-solving stage of Problem Follow-up (the importance of which was described in Chapter 1). In fact, both versions of the problem (PSA B and PSA B') have 18 different solutions. Although different children arrived at different solutions, the dismaying fact is that not one child in either group (nor any pilot-test subject) said anything about there possibly being more than one way to assign the guests or clocks. Apparently no one looked for another way, and certainly no one found another solution. We must acknowledge, of course, that the problem did not specifically ask for alternative solutions, and the mere presence of the interviewer in the room, waiting for the subject to finish, may have discouraged any search for other ways of doing the problem.

Nonetheless, one could conclude that a central tenet of school mathematics (and of standard education generally) -- that once you find any answer at all, you should go on to the next problem -- is so deeply ingrained by the fifth grade that even a problem with 18 different solutions is insufficient to invite further reflection.

Clearly there are formative implications here for future production of **SQUARE ONE TV**. We consider the Problem Follow-Up stage to be of central importance, for reasons already presented, but apparently it is not receiving sufficient attention in the series to have produced an effect.[34] An increase in the percentage of problem-solving segments in which some sort of Problem Follow-up occurs, possibly combined with more explicit commentary on the appropriateness of the strategy, may be called for.

Despite **SQUARE ONE TV**'s apparent lack of effect on Problem Follow-up, the general finding of this study is that the problem-solving actions and heuristics that are modeled in **SQUARE ONE TV** appear to be used by viewers under certain circumstances. However, the pattern of bars in Figure 5.9 in Chapter 5 is somewhat different for each of the three PSAs, and none of them precisely matches Figure 6.4's pattern of bars for Goal II subgoals represented in the 30 programs. Thus we can make no claim that the presence of a particular action or heuristic in the series produces a proportional effect in viewers; and, in light of the points made above, no such claim would be reasonable. Rather, the effect of **SQUARE ONE TV** on specific problem-solving actions and heuristics is a function of three factors: series content, problem demands, and children's natural inclinations.

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[34] Some form of Problem Follow-up (i.e. some subsubgoal of IID) was present in 56% of the 116 problem-solving segments contained in the 30 programs that the viewers watched. (In contrast, virtually all of the segments contained some form of Problem Formulation and Problem Treatment, and more than 86% contained one or more examples of Heuristics.) Across the 115 programs produced in Seasons I and II, Problem Follow-up occurs in 47% of the problem-solving segments. Thus our sample of 30 programs is somewhat more heavily weighted in Problem Follow-up than the first two seasons of the series as a whole.

Nonviewers' Performance

To this point, we have focused primarily on the performance of viewers. We had hypothesized that exposure to **SQUARE ONE TV** would result in viewers' using a greater number and variety of problem-solving behaviors in the posttest, and, because nonviewers would not be exposed to the treatment, we expected their performance to remain unchanged. These expectations were confirmed in that viewers did indeed use a greater number of problem-solving actions and heuristics in the posttest. But in PSAs A* and B*, we found that nonviewers tended to use fewer problem-solving actions in the posttest than they had in the pretest. Let us examine a few potential explanations for nonviewers' drop in performance, and consider each in turn.

One explanation is that nonviewers might have seen posttest problems as being similar to those presented in the pretest. In this case, they would be likely to use a subset of whatever they considered to have been the most effective behaviors in the pretest to try to solve them. If so, why were the viewers not affected in the same way? Two forces may be at work here: One is that the intervening mass of rich, problem-solving situations seen on **SQUARE ONE TV** might actually have lessened viewers' recall of the problems used in the pretest. Rather than approaching the posttest PSAs as just another version of something they had done before, then, viewers would treat the PSAs as fresh problems. Another force that may be operating, of course, is the generally salutary effect of watching **SQUARE ONE TV** discussed earlier. That is, even if viewers remembered the pretest PSAs just as well as nonviewers did, viewers had been exposed to numerous **SQUARE ONE TV** segments that explicitly say that (in the words of one "Mathnet" segment) "when you're trying to solve a problem, it's good to examine all the possibilities." Unlike nonviewers, then, viewers could have been sufficiently influenced by these messages to approach the PSAs from several angles, thus bringing new actions and heuristics to bear on the PSAs.

A second explanation for the nonviewers' drop in performance rests on children's attitudes towards the PSAs and mathematics in general. Consider the experience of the children in this study: They were taken from class for two one-hour sessions in which they were asked to solve a set of unfamiliar problems; then, eight weeks later, the process was repeated. It would not be unreasonable to expect that while children might be motivated to try to impress the interviewers and "do their best" in the pretest, their motivation might flag somewhat by the posttest. But this effect might be counterbalanced to some degree for the children who had been exposed regularly to **SQUARE ONE TV**, for the motivational reasons discussed earlier. (This is supported by the significant correlation between magnitude of motivation and P-scores.)

In examining these two explanations one should keep in mind that the nonviewers' P-scores dropped significantly only in PSAs A* and B*. Of the three PSAs, A* and B* are the least complex, and, perhaps more importantly, the similarity within each of those pairs (i.e., between the non-prime and prime versions of each PSA) is the strongest.[35] This observation lends plausibility to the first explanation, above, because, presumably, two versions of the same PSA are more likely to be perceived as similar if in fact they are similar.

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[35] Cf. Chapter 3 of this volume and Appendix II.D.

The Level of Viewers' Exposure to SQUARE ONE TV

Let us now turn back to the viewers in the present study, through a discussion of their level of exposure to **SQUARE ONE TV**.

Recall that the treatment in this experiment consisted of watching 30 programs of **SQUARE ONE TV**, with no teacher support, for one half hour each weekday over the course of eight weeks. The attendance rate during this period was very high, so we are confident that most of the children in the viewing group actually watched most of the programs. Several points should be made about this level of viewing, particularly vis-à-vis the general implications of this study.

We chose six weeks of viewing largely because the school district involved was able to accommodate this amount. Given the anticipated difficulty of changing children's problem-solving behavior and their attitudes toward mathematics, we would have preferred a longer treatment period, but we considered six weeks acceptable.

In the context of this study, which involved sustained, unaided viewing, one could argue that six weeks of daily viewing is fairly intensive. We know from other sources (e.g., Nielsen, 1988) that it is unusual for children to watch **SQUARE ONE TV** five days a week. On the other hand, the broadcast season of **SQUARE ONE TV** stretches over much more than six weeks, so it is entirely feasible for a child to watch 30 (or more) programs spread out over a longer period[36].

Ultimately, it does not really matter which perspective one adopts; rather, one must simply recognize that the study examines what is possible given exposure to 30 programs of SQUARE ONE TV under the conditions and using the instruments described. The results presented in **Chapter 5** indicate that this level of exposure can exert a profound effect. It is unclear how far the level of viewing could be reduced while maintaining the same results.

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[36] With the production of Season III, the **SQUARE ONE TV** library currently consists of 155 half-hour programs.

Conversely, one would imagine that if viewing were increased to a higher level, or were combined with appropriate instruction, the effects could be even greater. The implications of these questions for future study are discussed below.

Another important point to be made here is that the viewing group, by watching **SQUARE ONE TV**, spent an extra half hour daily on mathematics. A critic might remark, correctly, that the viewing time was not taken from time ordinarily devoted to mathematics instruction. In response, we note that **SQUARE ONE TV** is designed for a home audience; hence anyone watching the series at home is also spending an extra half hour on mathematics. Indeed, this is precisely the aim of the series. And our results certainly suggest that if one is interested in enhanced problem-solving performance, this is a half hour well spent.

Implications for Future Research

Given the nature of the treatment in the current study, there are a variety of future research studies that would be helpful to get a fuller picture of the effects of **SQUARE ONE TV** on children's problem solving. As examples, one might:

- conduct essentially the same study, but test at least some of the subjects at an intermediate point, perhaps after three weeks;
- increase or decrease the treatment period;
- conduct a longitudinal study of naturalistic home viewing, with careful measurements of outcomes; or
- systematically vary levels of teacher use of the series in classrooms, possibly providing some teachers with explicit training and materials to help them incorporate the series into their mathematics instruction.

Such studies would be valuable in shedding light on a variety of issues with respect to **SQUARE ONE TV**. Their results would be a worthwhile complement to the present study's finding that unaided viewing of 30 programs has a significant impact upon children's problem-solving behavior.

How Does This Study Relate to the Reform Movement?

To summarize our discussion of the results and implications of the present study, let us consider the relationships among school mathematics (including curriculum, instruction and evaluation), **SQUARE ONE TV**, and the PSAs -- all in light of the current reform movement.

SQUARE ONE TV vis-à-vis the PSAs. We have already explored in some detail (in Chapter 3 of this volume) the relationships between **SQUARE ONE TV** and the Problem-Solving Activities that were used to evaluate the effects of the series. Essentially we have argued that the PSAs were appropriate instruments to use, and that the methods of quantifying the subjects' performances on them were reasonable ones. Because the PSAs and their scoring system were explicitly aligned with the goals of the series, they provide a valid assessment of the extent to which one of the goals (Goal II) of **SQUARE ONE TV** has been achieved. And, in fact, the results reported in Chapter 5 indicate that **SQUARE ONE TV** can have a substantial positive effect on children's problem solving.

One set of findings is particularly noteworthy here, namely, the lack of significant interactions with sex, ethnicity, or SES. A major concern of the reform movement has been to increase the involvement of women and members of minority groups in the mathematical enterprise -- course enrollment, careers that use mathematical sciences, and so on (NRC, 1989). What this study suggests is that **SQUARE ONE TV** can exert a similar effect on the mathematical problem solving of boys and girls and children of different ethnic groups and socioeconomic backgrounds.

School mathematics vis-à-vis **SQUARE ONE TV**. Consider next the relation between the school mathematics curriculum and **SQUARE ONE TV**. Throughout this volume we have asserted that the match between mathematics in school and mathematics on **SQUARE ONE TV** is not particularly close.[37] **SQUARE ONE TV**'s content is considerably broader, and its emphasis on problem solving and attitudinal factors is deeper. This dissonance between school mathematics and **SQUARE ONE TV** mathematics is amplified by considering the next relationship -- the one between school mathematics and the PSAs.

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[37] Again, however, we stress that there are exceptions to this generalization.

School mathematics vis-à-vis the PSAs. A striking fact here is that there is little correlation between children's pretest scores on the PSAs and their scores on the standardized mathematics test that was administered to every fifth grader in the city.

From one viewpoint, of course, this lack of correlation is not at all surprising, simply because the PSAs are so different from standardized multiple-choice tests. Indeed, the two kinds of tests are designed to measure different things. The skills that are valuable in standardized tests -- notably the ability to select quickly the single correct response to a computational exercise -- are not needed in doing the PSAs[38]. Conversely, the kind of reflective, sustained investigation that leads to success on the PSAs may be of little use on a standardized test.

On the other hand, standardized tests and the PSAs both purport to measure something connected with mathematics, and hence one might expect a positive relation between them. From this standpoint, then, the lack of correlation is noteworthy, even if it is not surprising. In particular, it highlights the chasm between mathematics education as currently practiced and the vision of mathematics education described in the Standards (NCTM, 1989) and other documents of the reform movement.

Recall also that there is virtually no correlation between viewers' standardized test scores and the changes (from pretest to posttest) in their P-scores or M-scores. That is, the viewers in our sample seemed to benefit from watching **SQUARE ONE TV** regardless of their standardized test scores. Of course one cannot conclude from this that children of varying abilities benefit equally from **SQUARE ONE TV**, because the evidence seems to be that standardized test scores are not, in fact, particularly good measures of mathematical ability

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[38] The most difficult calculations required by the PSAs are 4× 8 and 5× 6, in PSA C and C', respectively. Computations of this sort posed no difficulty for any of the children in the main study (although a few of the pilot test subjects occasionally stumbled on them).

(NCTM, 1989). One can conclude, though, that **SQUARE ONE TV** does not have different effects on high and low performers on standardized tests.

The lack of correlations between standardized test scores, on the one hand, and P- and M-scores (or growth in P- or M-scores) on the other, can lead to one to adopt one of two positions. One position is this: Because viewing **SQUARE ONE TV** results in children's mathematical growth (as measured by the PSAs), but the PSAs have no correlation with success in school mathematics (as measured by the standardized tests), it then follows that the series may simply be disconnected from school mathematics.

An alternative position is this: The kinds of problem-solving activities that are represented by the PSAs are much closer to what is important in mathematics than are multiple-choice tests of computation and recall. Therefore, the lack of agreement between the PSAs and standardized tests suggests that the latter are not measuring what is central to mathematical behavior. Moreover, to the extent that their performance on the PSAs increases as a result of persistent viewing of **SQUARE ONE TV**, the children are making worthwhile mathematical gains, regardless of any relation (or lack of relation) between the series and the school curriculum.

Actually, the two positions are not incompatible. One can acknowledge the differences between **SQUARE ONE TV** and school mathematics as it is generally practiced today, while simultaneously recognizing the aspirations of the reform movement to change the practice of school mathematics.

If one agrees that the PSAs reflect genuine mathematical thinking and behavior more closely than multiple-choice instruments do, then instruments like the PSAs could be helpful in evaluating curricular improvements in school mathematics. In fact, tests of problem solving such as the PSAs could play a role in highlighting for teachers, children, parents and administrators some of the important, but now neglected, outcomes of mathematical instruction. Further, such instruments could provide the instructionally relevant feedback to all of those groups that norm-referenced testing now fails to serve.

The position one takes on these matters will depend, of course, on one's general conception of mathematics -- what it is and what children should know about it and be able to do with it. We feel confident that proponents of reform in mathematics education, as exemplified by the NCTM Standards document, would adopt the position that the lack of relation between standardized test scores and scores on the PSAs results primarily from the PSAs' ability to measure mathematically interesting and important behavior.

Summary

In conclusion, we recapitulate the points made in this volume. The first four chapters have described the conceptualization, development and pilot testing of instrumentation closely aligned (in the sense of NCTM's (1989) Standards) with the problem-solving goal of **SQUARE ONE TV**. The instruments were administered, as pretests and posttests, to a group of fifth graders, half of whom were exposed to 30 programs from the first two seasons of SQUARE ONE TV. The results, set forth in Chapter 5, indicate that viewing **SQUARE ONE TV** had a powerful positive effect on those viewers' problem solving, and that this effect was not different for children of differing sex, SES, ethnicity, or standardized test performance. Our final chapter has shown that we cannot pinpoint precise causal relationships between program content (on the subsubgoal level) and viewers' subsequent performance on the problem-solving tasks, particularly since attitudinal effects may also be involved. Nonetheless, an overall conclusion is apparent: sustained, unaided viewing of **SQUARE ONE TV** can have a significant impact on children's problem solving.

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APPENDIX II.A:

PSA-Specific Pilot Test Issues

This appendix describes pilot testing issues that were specific to each of the three PSAs. Readers may find it helpful to read this appendix in connection with the descriptions of the PSAs presented in Chapter 3.

PSA C*

PSA C* was developed first. We knew we wanted one problem to be fairly complex and difficult. Our first task was to determine what PSA C* should be. Over the course of the pilot phase we tried three candidates (in addition to the dice/spinner game problem, which was the one we finally selected) as the most difficult PSA[1], each of them with children of varying abilities. We concluded that one of them did not elicit sufficient variety of problem-solving approaches; another was too difficult, despite efforts at simplification; and results from the third were difficult to interpret. While further refinement might make any of them suitable for a study of children's problem solving, each was ultimately rejected in favor of the dice/spinner game problem.

Even when that choice had been made, there were a number of other issues that needed to be resolved:

Who should play the game? Should the players be the subject and interviewer, or should there be players dissociated from the subject and interviewer? Even though using dissociated players increases the complexity of the presentation of the game, even more complicating factors arise when the subject and the interviewer are the players (e.g., competitiveness, the subject's natural relationships with adults and strangers, inter-gender cross-socialization), and these factors made interpretation of results difficult. In the interests of increased reliability across interviewers and subjects, then, we decided that the players should be androgenous, cut-out, foam-core "players."

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[1] One of these problems was a geometric problem that involved placing as many 3" × 5" sandwiches as possible inside the borders of a 14" × 21" tray without overlap, and justifying that maximum. A second problem involved selecting questions that would help to identify the contents of four paper sacks, each containing a different combination of colored marbles. This task was designed so that one of the useful tools available was to draw a random sample from one of the bags. The third problem was to construct a model of a building using a given set of walls, doors and windows, subject to certain conditions, while maximizing floor area and minimizing cost.

Who determines what's wrong with the game? How much guidance should the subject receive regarding the mathematical crux of PSA C* (i.e. that it is unfair to one of the players)? We experimented with three different levels here: (1) telling the subject what is wrong with the game (so that the problem becomes only one of fixing the game); (2) telling the subject that something is wrong with the game without specifying what it is; and (3) telling the subject nothing about anything being wrong with the game, and letting him or her discover that the game is unfair. The middle level of guidance worked best from the viewpoint of balancing accessibility with challenge. Of course this choice meant that some children might find non-mathematical things wrong with the game -- equipment flaws, or lack of appeal to potential players, for example. This was perfectly acceptable since we wanted to see if children perceived the problem as one in which mathematics might be useful, and to see if these perceptions might change over time.

When should the game be fixed? A further question here was whether the subject should be asked to fix the game at the outset or only when he or she had first declared what wrong. We chose the latter course for two reasons, even though it required the experimenter to go away and return twice during the problem. The first reason was that presenting both parts of the problem at the beginning seemed unmanageable for some children, particularly in light of the overall complexity of the problem. Second, by breaking up the problem into two parts we increased the amount of time that could be devoted to exploring the child's use of probabilistic thinking. The concepts involved are by and large unfamiliar to children of this age (especially since probability is not stressed in the usual school curriculum), and thus we felt that having two opportunities to talk about it, rather than just one, would be time well spent.

PSA B*

This PSA grew from an abstraction of one part of PSA C*, i.e. the idea of separating an ordered set of numbers into subsets meeting certain conditions (see Appendix II.E on the mathematical relationships among the PSAs). Issues that were explored here included:

Dealing with decades. Children have a natural tendency to group numbered cards by decades. The numbers used were therefore chosen so that separating cards entirely by decades would yield the wrong number of groups and not be helpful at all in meeting the optional condition (see below).

Number of conditions. Would subjects be able to deal with the number and complexity of the conditions? We found that by repeating the requirements of the problem and providing the subjects with a list of them minimized memory demands and made the problem as a whole manageable.

Optional condition. After considerable experimentation we decided to make one of the conditions (that one pile should have exactly twice as many cards as one of the other piles) optional. One reason was to make the problem somewhat less daunting for some of the subjects who might have trouble with it. A second reason was that we thought it might allow us another window through which to view matters related to motivation.

PSA A*

This was designed to be even simpler than PSA B*, though with some connection with part of PSA C*.

Number of objects. Should there be three objects or four? After experimenting with both three and four objects it became apparent that three was too easy -- the subject could find all six orders without using any systematicity, either in generating orders or keeping track of them. As a result, both versions of this PSA use four objects.

APPENDIX II.B:

Protocols for the Problem-Solving Activities

This appendix presents the protocols used in administering the PSAs in the pretest. Recall from Chapter 3 that children completed one set of PSAs (A, B and C; or A', B' and C') in the pretest and the other set in the posttest.

The posttest protocols for PSAs A, A', B, and B' are identical to those shown here. The posttest protocols for PSAs C and C' differ from the pretest protocols only in that they begin, "Remember from the last time there was an owner of a game factory, Dr. Game? He was recently very upset..."

PRETEST
SESSION 1 - PSAs C'and B'

This protocol is for use in the pretest, session 1, PSAs C' & B' (prime set).

NOTE:
As highlighted in the protocol, you have:
- MAXIMUM 40 MINUTES -- PSA C'
- MINIMUM 15 MINUTES -- PSA B'
- THE ENTIRE SESSION CANNOT EXCEED 55 MINUTES

Before we start, I'd like to say that what we are interested in is how you think about the things that we are going to do today. At the end of each thing that I ask you to do, I will ask you lots of questions about what you did and what you were thinking. So, if you could try to remember what you are thinking as you are doing these different things, that would be really helpful. And when I ask you these questions at the end, I really do want to know everything that you thought -- your good ideas, your bad ideas and ideas that you think don't matter or aren't important. Is that clear? Any questions? Okay, let's begin.

PSA C'

Before I explain what I'd like you to do today, I'd like to give you some background information. The owner of a game factory, Dr. Game, was recently very upset to discover that his factory had been broken into. His arch enemy, Mr. Enemy, got into several of his games and made something wrong with them -- changed them in some way.

You've been hired to help find out what it is that is wrong with one of the games that Mr. Enemy has changed. I will show you the pieces for the game and tell you how it works. Then it's your job to find out what is wrong with it.

Here are two SPINNERS for the game. Here is a coin. Why don't you take a look at these. [Hand child spinners and coin. Give him or her some time to examine them, and discuss only if child initiates discussion. Keep discussion here brief.]

Here's the game. There are two players -- a green player and an orange player [show cut-out figures]. There is also a spinner-person. The spinner-person spins both of the spinners [spin spinners] and gets two numbers, one on this spinner and one on this spinner. If it lands on a line, the spinner-person just spins again. Then the spinner-person flips the coin and gets a "plus" or a "times" [show plus and times and flip coin]. Then the spinner-person does whatever is on the coin -- the addition or multiplication, and finds the answer here [show chart]. If the answer is inside the orange loop, then the orange player gets a chip, and if the answer is inside the green loop, then the green player gets a chip. There are nine chips here. Whoever has more chips at the end of nine spins of the spinners wins the game. Those are the rules. Does this make sense so far?

Here's an example. Suppose this spinner comes up with a 2 and this spinner is 4 [rotate spinners so that they point to 2 and 4 respectively], and the coin is "plus" [turn coin so that "plus" is up]. You do the addition -- 2 plus 4 is 6. Then you look to find 6 on this [point to chart]. It's here, inside the orange loop [point to the orange band], so the orange player wins a chip. [Move a chip so that it is in front of the orange player.] Do you get it?

Let's try another example. [Remove the chip that is in front of the orange player.] Say this spinner shows a 6 and the other spinner says 3; and suppose the coin comes up "times" [move spinners and coin appropriately]. Who wins a chip this time? [Green, because 6 x 3 is 18, which is inside the green loop. Move chip to front of green player briefly, and then return it to the pile of chips.]

Remember, there's a bunch of stuff here for you to use if you'd like. If you want anything, just help yourself. You may want to use something and you may not -- it's up to you.

Again, what I said before is that there's something wrong with this game and you've been hired to find out what is wrong. You should know before you start thinking about this that Dr. Game took a quick glance at the game and was relieved to notice that none of the pieces were missing. But there is still something wrong with the game.

I'm going to go over there while you're working on this. Why don't you take a little while to think about this. Don't rush, take as much time as you like. If you need something, or need to ask me any questions, let me know. Then, when you think you've found out what is wrong with the game, let me know and I'll come back. Then we can talk about what you think. [Go to other side of the room].

CHECK SUBJECT AT 10 MINUTES TO SEE HOW PROGRESSING.
GIVE SUBJECT A MAXIMUM OF 15 MINUTES BEFORE RETURNING.

WHEN CHILD IS FINISHED : Return to testing area. Initiate discussion of what subject has done.

Follow-up probes:
>>> What do you think is wrong with the game? [standard probes]

NOTE: If child tries to engage you to help solving the problem by asking you questions, say "What do you think?"

>>> How do you know that? [standard probes]

>>> Okay. I wasn't here while you were doing this. I want to know what you were doing and what you were thinking.

So what did you do first and what were you thinking? [standard probes]
Then what did you do? What were you thinking then? [standard probes]
Then what did you do? ... [continue until child is finished]

>>> One last question. One person who worked on this game thought, at first, that the problem had something to do with how much the coin weighs. But then he/she thought about it, and realized that the weight wasn't important. You see, he/she had an idea, but then he/she realized that that idea wasn't helpful. Did anything like that happen to you while you were working on this? That is, did you try or think about anything that you later realized wasn't helpful? [standard probes.]

WHEN DISCUSSION IS FINISHED:

Alright, now, I'd like you to try something else. You've told me what you think is wrong with the game. Now, Dr. Game wants to hire you to fix the game. Again, I'm going to go over there while you are working on this. If you need anything, or want to ask me any questions, go right ahead. Take as much time as you like. There's no need to rush. When you think you've come up with a way to fix the game, let me know. I'll come back and we'll discuss what you have done. [Go to other side of room.]

SEE HOW MUCH TIME LEFT.
ALLOW SUBJECT A MAXIMUM OF 40 MINUTES TO COMPLETE BOTH PARTS ("WHAT'S WRONG" AND "FIX IT").

WHEN CHILD IS FINISHED: Return to testing area. Initiate discussion of what child has done.

Follow-up probes:
>>>So, did you fix the game? [standard probes]

>>>How does that fix it? [standard probes]

>>>Okay. I wasn't here while you were doing this. I want to know what you were doing and what you were thinking.

So what did you do first and what were you thinking? [standard probes]
Then what did you do? What were you thinking? [standard probes]
Then what did you do? [continue probing until child is finished]

>>>Did you think about anything while you were working on this that you later realized wasn't helpful? [standard probes]

ALLOW A MINIMUM OF 15 MINUTES FOR THIS PORTION OF THE INTERVIEW.

PSA B'

I've got something else here that I'd like you to do.

Before we start , I want to be sure that you know what "twice as many" means. "Twice as many" means the same thing as "two times as many" or "double". For example, what's twice as many as 10? [If there is any hesitation, or S gives a wrong answer, ask "What's two times ten?" If there's still a problem, say "Two times ten is twenty, so twice as many as 10 is 20."]

What's twice as many as 20? [Supply answer if needed. Agree with S's answer if correct.]

Can you make up your own example? You pick a number. What's twice as many (as the number S picked)?

[Once child seems to have grasped concept, proceed with interview.]

I also want to be sure that you know what the word "optional" means.
Any ideas?
When something is optional, it means that you don't have to do it. You can do it if you want to--it's up to you. You have a choice.

[Once child seems to have grasped this concept, proceed with interview.]

Let's pretend that you run a store that sells different kinds of high quality clocks. I don't have the clocks with me, but I do have some price tags. [Show the S the tags, but DO NOT give them to S yet.] [Show two different tags for clocks of different prices.] Like this is a ___________ clock, and its price is _______ dollars. This is a __________ clock, and it costs __________ dollars. Some clocks have the same price, but most of them do not.

You want to set up a special display case for these clocks. The display case has three shelves. You want there to be a shelf for the more expensive clocks, a shelf for clocks with middle prices, and a shelf for the less expensive clocks. Each clock on the more expensive shelf has to cost more than every clock on the other shelves, and each clock on the less expensive shelf must cost less than every clock on the other shelves.

Also, there should be at least four clocks on each shelf. You can have more than four on a shelf if you want to, but there must be at least four.

There's one other thing: this is optional. You don't have to do this, but if you want to, you can try to make one of the three shelves have exactly twice as many -- that's two times as many -- clocks as one of the other shelves. The third shelf can have whatever is left.

Now, that was sort of complicated, so I'll repeat it and give you this [show chart and point while reading]:

1. Assign all the clocks to three shelves by making three piles of price tags.
2. Make a shelf for more expensive clocks, one for middle-priced clocks, and one for the less expensive clocks.

3. Each shelf must have at least four clocks.

[After reading #3 say: "Of course you can have more than four clocks on a shelf, but each shelf must have at least four. What I mean is, there must be four or more clocks on each shelf.")

OPTIONAL: Make one of the three shelves have exactly twice as many clocks as one of the other shelves.

Remember, the last part is optional.

When you have assigned each clock to a shelf by making one pile of price tags for each shelf, let me know, and we can talk about what you have done.

Does this make sense? Do you have any questions?

[Note that the optional condition, above, doesn't put any restrictions on one of the shelves, so if S asks something like "What about the third shelf?" the interviewer can say "You can have as many clocks there as there needs to be."]

[HAND OUT PRICE TAGS to the child.]

I'm going to go over there while you're working on this. If you need anything, or need to ask me any questions, let me know. And don't forget, there's a bunch of stuff here for you to use if you'd like. When you're finished, tell me, and I'll come back. Now, I may have to interrupt you before you're finished. That's okay. At that point, we'll talk about what you've been doing and what you're thinking. As I said before, I'm interested in how you think about things. [Go to other side of room.]

DEPENDING ON TIME, GIVE SUBJECT 5-8 MINUTES TO WORK ON THIS BEFORE RETURNING

WHEN TIME HAS ELAPSED/SUBJECT IS FINISHED: Return to testing area. Initiate discussion of what child has done.

Follow-up probes:
>>>Can you tell me what you did?

>>>Okay. I wasn't here while you were doing this. I want to know what you were doing and what you were thinking.
So what did you do first and what were you thinking? [standard probes]
Then what did you do? What were you thinking? [standard probes]
Then what did you do? ... [continue until child is finished]

>>> [If child has not provided detailed explanation of what they have done, ask the following series of questions:]

1. **Did you make three piles?**
   Can you show me?

2. **Did you make a shelf for more expensive clocks, one for middle-priced clocks, and one for the least expensive clocks?**
   How do you know? [If have not received satisfactory answer to the "how do you

know" question, say: "Can you show me that all the clocks in the most expensive pile are more expensive than all the other clocks?"]

3. Are there at least four clocks on each shelf?
   Can you show me?

4. I know I said this was optional, but did you try to make one of the three shelves have exactly twice as many clocks as one of the other shelves?
   IF YES: Could you do it?
   Can you show me?
   IF NO: How come?
   Do you think it's possible? Why or why not?

>>>Did you think about anything while you were working on this that you later realized wasn't helpful?, e.g. someone thought that he/she should group the clocks by alphabetical order, but then realized that that wasn't helpful. Did anything like that happen to you? That is, did you try or think about anything that you later realized wasn't helpful? (standard probes)

That's all the questions I have for you. Do you have any questions for me?
Thank you very much for helping us.
Someone else will be here to see you tomorrow. You have your card, right? Okay, bye-bye.

PRETEST
SESSION 2 - PSA A' and Interview

This protocol is for use in the pretest, session 2, PSA A' & Attitude Interview(prime set).

As you know, what we're interested in is how you think about things. So, I'll be asking you all sorts of questions about how you think and feel.

[The interview starts with several questions concerning the child's experiences on the previous day with PSAs C' and B'.]

I have another thing here.

[Say "SHE/HER" with GIRLS and "HE/HIS" with BOYS]
[Place open card holder on table in front of child.]

A circus owner is about to take her show on the road and go touring around the country for a while. Her circus has only four performers, and here they are: [show each picture as you name it and lay them on the table from child's left to right]: a juggler, an acrobat, a magician and a clown.

Each person here [gesture to pictures] will perform every night, and the circus owner wants them to perform in as many orders as possible. One order, for example, is [pick up cards and place in card holder from left to right] juggler, magician, acrobat and clown. This means that on this night, [point to each in turn], the juggler will perform first, the magician will go second, the acrobat will be third, and the clown will be fourth. Can you show me another order? [Let child show you another order. If child seems to have difficulty grasping concept, repeat explanation of problem.]

Now, it's your job to find the total number of orders that the circus owner can make.

I'm going to go over there while you are working on this. When you think you've come up with the largest number of orders, and are sure that you haven't repeated any of them, let me know, and I'll come back. Then we can discuss what you have done.

Now, I may have to interrupt you before you finish, but that's okay. At that point, we'll talk about what you've been doing and what you're thinking. As I said before, I'm interested in how you think about things. Don't forget, there's a bunch of stuff here for you to use if you'd like. [Go to other side of room.]

ALLOW THE SUBJECT 5 - 8 MINUTES TO COMPLETE THIS TASK

IF YOU'VE INTERRUPTED THE CHILD:

If you had more time to work on this, what would you do?
How many orders did you make? How many do you think you could have gotten?
How do you know that you haven't repeated any so far?
Do you think that somebody else might be able to find other orders than you?

IF THE CHILD HAS FINISHED:

Okay. How many orders did you make?

Are you sure that you've made the most possible orders? How do you know that you have them all?

Are you sure that you haven't repeated any of them? How do you know you haven't repeated any?

Do you think that somebody else might be able to find other orders?

So, I wasn't paying attention while you were working on this. Could you tell me what you were doing and what you were thinking? What did you do first and what were you thinking?

Then what did you do? What were you thinking?

Then what did you do? [cont. until child is finished]

Did you realize or discover anything when you were doing this?

[The session continues with the Attitude Interview. For the Attitude Interview, see Volume III.]

PRETEST
SESSION 1 - PSAs C and B

This protocol is for use in the pretest, session 1, PSAs C & B (non-prime set).

NOTE:
As highlighted in the protocol, you have:
-MAXIMUM 40 MINUTES -- PSA C
-MINIMUM 15 MINUTES -- PSA B
-THE ENTIRE SESSION CANNOT EXCEED 55 MINUTES

Before we start, I'd like to say that what we are interested in is how you think about the things that we are going to do today. At the end of each thing that I ask you to do, I will ask you lots of questions about what you did and what you were thinking. So, if you could try to remember what you are thinking as you are doing these different things, that would be really helpful. And when I ask you these questions at the end, I really do want to know everything that you thought -- your good ideas, your bad ideas and ideas that you think don't matter or aren't important. Is that clear? Any questions? Okay, let's begin.

PSA C

Before I explain what I'd like you to do, I'd like to give you some background information. The owner of a game factory, Dr. Game, was recently very upset to discover that his factory had been broken into. His arch enemy, Mr. Enemy, got into several of his games and made something wrong with them -- changed them in some way.

You've been hired to help find out what it is that is wrong with one of the games that Mr. Enemy has changed. I will show you the pieces for the game and tell you how it works. Then it's your job to find out what is wrong with it.

Here are the two dice for the game. Here is a coin. Why don't you take a look at these. [Hand child dice and coin. Give him/her some time to examine and discuss only if child initiates discussion. Keep discussion brief here.]

Here's the game. There are two players -- a RED player and a PURPLE player [show cardboard figures]. There is also a dice thrower. The dice thrower rolls both of the dice and gets two numbers, one on this die and one on this die [roll dice]. Then the dice thrower flips the coin and gets a "plus" or a "times" [flip coin and show plus and times]. Then the dice thrower does the addition or multiplication. If the answer is less than 10, then the RED player gets one chip [affix "less than 10" sign to RED player]. If the answer is 10 or more, then the PURPLE player gets one chip [affix "10 or more" sign to PURPLE player]. There are 9 chips here. Whoever has more chips at the end of 9 rolls of the dice wins the game. Those are the rules. Does this make sense so far?

Here's an example. Say this die [octahedron] comes up with a 5, and this one [cube] comes up with a 3, and the coin is a "plus." Then the RED player would win a chip because 5 + 3 is 8, and 8 is less than 10. [Move a chip so that it is in front of the red player.] Do you get it?

Okay, let's try another example. Say this die [octahedron] comes up with a 5 and this one [cube] comes up with a 2 and the coin says "times." Who wins a chip this time? [PURPLE, because 5x2 is 10. Move chip in front of PURPLE player briefly, and then return it to the pile of chips. If child is confused, make sure he understands the meaning of "10 or more."]

Remember, there's a bunch of stuff here for you to use if you'd like. If you want anything, just help yourself. You may want to use something and you may not--it's up to you.

Again, what I said before is that there's something wrong with this game and you've been hired to find out what is wrong. You should know before you start thinking about this that Dr. Game took a quick glance at the game and was relieved to notice that none of the pieces were missing. But there is still something wrong with the game.

I'm going to go over there while you're working on this. Why don't you take a little while to think about this. Don't rush, take as much time as you like. If you need something, or need to ask me any questions, let me know. Then, when you think you've found out what is wrong with the game, let me know and I'll come back. Then we can talk about what you think. [Go to other side of the room].

CHECK SUBJECT AT 10 MINUTES TO SEE HOW PROGRESSING
GIVE SUBJECT A MAXIMUM OF 15 MINUTES BEFORE RETURNING.

WHEN CHILD IS FINISHED : Return to testing area. Initiate discussion of what subject has done.

Follow-up probes:
>>> What do you think is wrong with the game? [standard probes] NOTE: If child tries to engage you to help solve the problem by asking you questions, say "What do you think?"

>>> How do you know that? [standard probes]

>>> Okay. I wasn't here while you were doing this. I want to know what you were doing and what you were thinking.
So what did you do first and what were you thinking? [standard probes]
Then what did you do? What were you thinking then? [standard probes]
Then what did you do? ... [continue until child is finished]

>>> One last question. One person who worked on this game thought, at first, that the problem had something to do with the weight of the coin. But then he/she thought about it, and realized that the weight wasn't important. You see, he/she had an idea, but then he/she realized that that idea wasn't helpful. Did anything like that happen to you while you were working on this? That is, did you try or think about anything that you later realized wasn't helpful? [standard probes.]

WHEN DISCUSSION IS FINISHED:

I'd like you to try something else. You've told me what you think is wrong with the game. Now, Dr. Game wants to hire you to fix the game. Again, I'm going to go over there while you are working on this. If you need anything, or want to ask me any questions, go right ahead. Take as much time as you like. There's no need to rush. When you think you've come up with a way to fix the game, let me know. I'll come back and we'll discuss what you have done. [Go to other side of room.]

SEE HOW MUCH TIME IS LEFT.
ALLOW SUBJECT A MAXIMUM OF 40 MINUTES TO COMPLETE BOTH PARTS ("WHAT'S WRONG" AND "FIX IT")

WHEN CHILD IS FINISHED: Return to testing area. Initiate discussion of what child has done.

Follow-up probes:

>>>So, did you fix the game? [standard probes]

>>>How does that fix it? [standard probes]

>>>Okay. I wasn't here while you were doing this. I want to know what you were doing and what you were thinking.
So what did you do first and what were you thinking? [standard probes]
Then what did you do? What were you thinking? [standard probes]
Then what did you do? [continue probing until child is finished]

>>>Did you think about anything while you were working on this that you later realized wasn't helpful? [standard probes]

LEAVE A MINIMUM OF 15 MINUTES FOR THIS PORTION OF THE INTERVIEW.

PSA B
I've got something else here that I'd like you to do.

Before we start, I'd like to be sure that you know what twice as many means. "Twice as many" means the same thing as "two times as many" or "double". For example, what's twice as many as 10? [If there is any hesitation, or S gives a wrong answer, ask "What's two times ten?" If there's still a problem, say "Two times ten is twenty, so twice as many as 10 is 20."]

What's twice as many as 20? [Supply answer if needed. Agree with S's answer if correct.]

Can you make up your own example? You pick a number. What's twice as many [as the number S picked]?

[Once child seems to have grasped concept, proceed with interview.]

I also want to be sure that you know what the word "optional" means.
Any ideas?
When something is optional, it means that you don't have to do it. You can do it if you want to--it's up to you. You have a choice.

[Once child seems to have grasped this concept, proceed with interview.]

There is going to be a party. The name and age of each person who is coming is written on one of these cards. [Show the S cards, but DO NOT give them to S yet.]
[Show two different cards of people with different ages] For example, _______ is invited, and he is _______. _______ is also invited, and she is _________. Some of the people are the same age, but most of them are not.

There are three tables. There's going to be a younger table where the younger people will sit, a middle table for people with ages in the middle, and an older table for the older people. Everyone at the older table must be older than everyone at any other table, and everyone at the younger table must be younger than everyone else at any other table.

Also, there should be at least five people at each table. You can have more than five at a table if you want to, but there must be at least five.

There's one other thing: this is optional. You don't have to do this, but if you want to, you can try to make one of the three tables have exactly twice as many -- that's two times as many -- guests as one of the other tables. That means that one table must have double the number of people as one other table. The third table can have whatever is left.

Now, that was sort of complicated, so I'll repeat it and give you this [show chart and point while reading]:

1. Assign all the guests to three tables by making three piles of cards.
2. Make a younger table, a middle table, and an older table.
3. Each table must have at least five guests.

[After reading #3 say: "Of course you can have more than five at a table, but each table must have at least five. What I mean is, there must be five or more guests at each table."]

> OPTIONAL: Make one of the three tables have exactly twice as many guests as one of the other tables.

Remember, the last part is optional.

When you've assigned each person to a table by making one pile of cards for each table, let me know, and we'll talk about what you've done.

Does this make sense? Do you have any questions?

[Note that the optional condition, above, doesn't put any restrictions on one of the tables, so if S asks something like "What about the third table?" the interviewer can say "You can have as many people there as there needs to be."]

[HAND OUT THE CARDS TO THE CHILD.]

I'm going to go over there while you're working on this. If you need anything, or need to ask me any questions, let me know. And don't forget, there's a bunch of stuff here for you to use if you'd like. When you're finished, tell me, and I'll come back. Now, I may have to interrupt you before you're finished. That's okay. At that point, we'll talk about what you've been doing and what you're thinking. As I said before, I'm interested in how you think about things. [Go to other side of room.]

DEPENDING ON TIME, GIVE SUBJECT 5-8 MINUTES TO WORK ON THIS BEFORE RETURNING

WHEN TIME HAS ELAPSED/SUBJECT IS FINISHED: Return to testing area. Initiate discussion of what child has done.

Follow-up probes:

>>>Can you tell me what you did?

>>>Okay. I wasn't here while you were doing this. I want to know what you were doing and what you were thinking.
So what did you do first and what were you thinking? [standard probes]
Then what did you do? What were you thinking? [standard probes]
Then what did you do? ... [continue until child is finished]

>>> [If child has not provided detailed explanation of what they have done, ask the following series of questions:]

1. Did you make three piles?
   Can you show me?

2. Did you make a younger table, a middle table and an older table?
   How do you know? [If have not received satisfactory answer to the "how do you

know" question, and say: "Can you show me that all the guests at the older table are older than all the other guests?"]

3. Are there at least five guests at each table?
   Can you show me?

4. Even though it was optional, did you try to make one of the three tables have exactly twice as many guests as one of the other tables?
   IF YES: Could you do it?
   Can you show me?
   IF NO: How come?
   Do you think it's possible? Why or why not?

>>>Did you think about anything while you were working on this that you later realized wasn't helpful?, e.g. someone thought that he/she should group the people by alphabetical order, but then realized that that wasn't a good idea and wasn't helpful. Did anything like that happen to you? That is, did you try or think about anything that you later realized wasn't helpful? [standard probes]

Well, that's all that I have to ask you for now. Do you have any questions that you'd like to ask me? Thank you very much. someone else will be here to see you tomorrow. You have your card, right? Okay, bye-bye.

PRETEST
SESSION 2 - PSA A and INTERVIEW

NOTE: This protocol is for use in the pretest, session 2, PSA A and Attitude Interview.

As you know, what we're interested in is how you think about things. So, I'll be asking you all sorts of questions about how you think and feel.

[The interview starts with several questions concerning the child's experiences on the previous day with PSAs C and B]

I have another thing here.

[Say "SHE/HER" with GIRLS and "HE/HIS" with BOYS]

A clothing designer has only four colors that she can use in her new shirt design: blue, pink, yellow and grey [show stripes of colors].
This is the style of shirt [show shirt].

In each one of these stripe sections [point to each one of the four stripe sections], she must put one of these colors [point to each of the four colors]. She wants to make as many different versions of this shirt as possible, using as many different color orders as she can. One color order, for example, is [take color strips and place them in the shirt drawing from top to bottom] blue, yellow, pink and grey. Can you show me another order? [Let child show you another order. If child seems to have difficulty grasping concept, repeat explanation of problem.]

Now, it's your job to find the total number of color orders that the clothing designer can make.

I'm going to go over there while you are working on this. When you think you've come up with the largest number of orders, and are sure that you haven't repeated any of them, let me know, and I'll come back. Then we can discuss what you have done.

Now, I may have to interrupt you before you finish, but that's okay. At that point, we'll talk about what you've been doing and what you're thinking. As I said before, I'm interested in how you think about things. Don't forget, there's a bunch of stuff here for you to use if you'd like. [Go to other side of room.]

ALLOW THE SUBJECT 5 - 8 MINUTES TO COMPLETE THIS TASK

IF YOU'VE INTERRUPTED THE CHILD:

If you had more time to work on this, what would you do?
How many orders did you make? How many do you think you could have gotten?
How do you know that you haven't repeated any?
Do you think that somebody else might be able to find other orders than you?

IF THE CHILD HAS FINISHED:

Okay. How many orders did you make?
Are you sure that you've made the most possible orders? How do you know that you

have them all?
Are you sure that you haven't repeated any of them? How do you know you haven't repeated any?
Do you think that somebody else might be able to find other orders?

So, I wasn't paying attention while you were working on this. Could you tell me what you were doing and what you were thinking? What did you do first and what were you thinking?
Then what did you do? What were you thinking?
Then what did you do? [cont. until child is finished]

Did you realize or discover anything when you were doing this?
[The session next continues with the Attitiude Interview. For the Attitude Interview, see Volume III.]

APPENDIX II.C:

Interviewing Guidelines and Standard Probes

INTERVIEWING GUIDELINES AND STANDARD PROBES

Interviewing Guidelines:

1. Always phrase probes in terms of your own confusion and misconceptions.

2. During the introductions to the problems, repeat rules and/or concepts (e.g. "twice as many") as much as necessary to get things going.

   During discussion of problem, if child indicates misunderstanding of rules and/or concepts (e.g. thinks "twice as many" is "more than"), do not try to reteach rules/concepts. Only reteach if child explicitly asks for clarification.

3. Never use the words "problem" or "chart" in your explanation of any of the PSAs. If child uses these words and you want to probe re: his/her use of them, then you can use these words.

4. Never say "this" or "that" when referring to any of the props or materials. Always use the proper noun to which you are referring; this will help clarify action to the reader of a transcript.

   In general, try to use full nouns and not pronouns. Again, this is for clarity purposes, both for the subject and the coder.

5. Always begin interview by introducing self and others in the room and by saying, "Thanks for coming to help us today." Always end interview with "Thanks a lot. That was a big help. I appreciate it." Do not end with "That was great, excellent, extraordinary, etc."

6. If child asks as a point of information (not necessarily as an explanation of what is wrong with the game) any question pertaining to the physicial aspects of the game e.g. "Why is this man purple?", "Why do the numbers repeat?" respond simply by saying "I'm not sure." If child asks as a point of information any question that pertains to the outcome or workings of the game, e.g. "Why does one man always win?", "Why do all the numbers come out on one side?" respond with probe # 18: "Well, what do you think?"

7. Always make sure microphone, charts, etc. are not blocking camera's view of child's activity, and that charts, protocols, etc. are not blocking tape recorders/microphones. Note that the chart for PSA C' should lie flat.

8. Make sure that you point out presence of material kit at each of your departure points (PSA C* (what's wrong and fix), PSA B*, and PSA A*).

9. If during the "What's Wrong" part of PSA C*, child comes in immediately following explanation of task (while researcher is still at table or within 1 minute of researcher's departure from table) with "I know what's wrong...", state the following: "I'd like you to take some time to think about this...don't tell me yet...I'm going to go over there...when you've taken some time to think about this you can call me back, ok."

   (Note that if child's perception of what is wrong with the game is very superficial, s/he still has the fix it part of the interview to explore what is wrong.)

If during the "Fix It" part of PSA C* child comes in immediately following explanation of the task (while researcher is still at table or within 30 seconds of researcher's departure) with a way to fix, e.g. "Change the numbers..." state the following: "You can use anything over there that you'd like. I'd like you to take some time to work on this...don't tell me yet."

10. In PSA C*: If child is definite that there is nothing wrong with the game or has no idea what is wrong with the game, do not ask him or her to fix it. If child has a vague idea about what might be wrong with the game, ask him or her to fix it.

11. Make sure that in PSA C* you flip coin, roll dice and spin spinners.

12. Do not hand out cards for PSAs B* or paraphernalia for PSAs A* until you are finished providing explanation of problem.

13. If child asks if s/he was "right", ask probe #18: "Well, what do you think?" If child is still not satisfied, say the following: "There are lots of ways to do this. We're not interested in right or wrong answers."

Standard Probes:

1. Repeat original question.
2. How come?
3. Uh huh.
4. Okay.
5. What do you mean?
6. I'm sorry, I don't think I got that.
7. Could you explain that to me?
8. I don't understand./I'm not clear on that.
9. Could you say a little more about that?
10. Could you tell me what you mean by direct quote ?
11. I don't understand what you mean by direct quote.
12. When you say direct quote, what do you mean?
13. And how do you know that?
14. (Could you) Say that again?
15. Why was it direct quote?

16. What made it direct quote?

17. Anything else?

18. Well, what do you think?

19. Repeat what child says in question form.

20. How/Why does that make it wrong?

21. What's wrong with that?

22. Can you show me (how that works)?

23. Explain to me how what you did fixes it.

24. What do you think?

25. I want to make sure I understand you. When you said direct quote, what did you mean?

APPENDIX II.D:

Descriptions of Correspondences Between the PSAs: "Primes" and "Nonprimes"

DESCRIPTIONS OF CORRESPONDENCES BETWEEN THE PSAs: "PRIMES" AND "NON-PRIMES"

The purpose of this Appendix is to display the correspondences between each pair of PSAs (PSA A and A'; PSA B and B'; and PSA C and C') -- that is, to show how each object or relation in PSA A, B, or C relates to something in PSA A', B', or C', respectively. These corresponding features are placed symmetrically around the center line in what follows. We also note aspects of the PSAs that do not correspond.

PROBLEM-SOLVING ACTIVITY A*

PSA A		PSA A'
	To place 4	
colored stripes		circus perfomers
	in as many ways as possible	
vertically		horizontally
	on a	
shirt		circus program

Aspects of the two versions of the problem that do not correspond include:

The order of the stripes is purely spatial	While each order of performers on the program is spatial, it indicates a temporal order of performance
The outlined shirt and the stripes themselves are relatively plain	The pictures of the performers are colorful and relatively complex

PROBLEM SOLVING ACTIVITY B'

PSA B		PSA B'
	To assign	
26		23
	cards, each one bearing	
a person's name		the brand name of a clock
	and	
the person's age		the price of the clock
(36,37,37,40,41,43,45, 48,49,52,53,54,56,56, 57,58,59,62,63,64,65, 66,66,67,67,68)		(28,29,31,31,35,37,38, 40,42,44,46,47,48,49,50 51,52,53,54,57,61,63,67)
	to three	
tables		shelves
	so that each	
table		shelf
	has at least	
5 people		4 clocks
	and so that there is a	
table for the younger, middle, and older people		shelf for the less expensive, middle-priced, and more expensive clocks
	and, optionally, so that there are exactly twice as many	
people at		clocks on
	one	
table as at		shelf as on

one of the other

tables.

shelves.

Aspects of the two versions of the problem
that do not correspond include:

The cards are square and made
from relatively thick cardboard;
this facilitates arranging them
in rectangular arrays.

The cards are price tags made
of relatively thin cardboard.

PROBLEM-SOLVING ACTIVITY C'

PSA C	PSA C'
To determine what is wrong with a game in which	
one octahedral die labeled alternately 5 and 8	one spinner divided in sixths labeled alternately 2 and 6
and	
one cubical die labeled 2, 3, and 4 on pairs of opposite faces	one spinner divided in sixths labeled 3, 4, and 5 cyclically
are	
rolled	spun
and a coin labelled + and × is flipped; whereupon one chip is awarded to the	
Red	Orange
player if the sum or product is	
less than 10	in an orange loop containing 1 through 15
and one chip is awarded to the	
Purple	Green

if the sum or product is

10 or more

in a green loop containing 16 through 30

until 9 such

rolls

spins

have been made; at which point the player with more chips wins the game.

The second part of the problem is to fix whatever was found to be wrong in the first part.

The tables showing the 12 equally likely outcomes are these:

+	2	3	4
5	7	8	9
8	10	11	12

+	3	4	5
2	5	6	7
6	9	10	11

×	2	3	4
5	10	15	20
8	16	24	32

×	3	4	5
2	6	8	10
6	12	24	30

The winning combinations for

Red

Green

are shaded; the probability of

Purple

Orange

winning any chip is .75; hence

Purple

Orange

is heavily favored (P > 0.95) to win the whole game.

Aspects of the two versions of the problem that do not correspond include:

Not all faces of the dice are visibile simultaneously, which may promote manipulation of the equipment.	All the spinner numbers are visible simultaneously, although some turning of the pointer may be needed.
No set of possible sums and products is directly suggested by the equipment.	The board with the numbers 1 through 30 suggests a particular set of outcomes.

APPENDIX II.E:

The Mathematical Relationships Among the PSAs

ON THE MATHEMATICAL RELATIONSHIPS AMONG THE PSAS

Note: The connections between the PSAs in any pair (e.g., PSA A and PSA A') are already clear; so to simplify the notation below we will use "PSA A*", for example, as an abbreviation for "PSAs A and A'".

There are essentially two tasks involved in PSA C*:

(A) One task is to determine the set of possible weighted outcomes -- that is, the set of sums and products counted according to how many times they occur. This is the task both with the original numbers and operations, and also with whatever new numbers or operations (if any) the subject creates.

(B) The second task is to create a partition of that set of outcomes meeting a certain criterion. (A partition is simply a family of subsets of the original set that are pairwise disjoint (i.e. no two of them have anything in common) and together cover the whole set (i.e. everything in the original set is in one of the subsets).) In the case of PSA C*, the partition consists of two subsets, one for each of the two players. The condition they must meet is one of fairness, which means having the same number of elements in each (weighted by the probabilities of their occurrences).

The mathematical underpinnings of these two tasks are shared with the other PSAs: Task (A) with PSA A*, and Task (B) with PSA B*. In addition, there is a further connection between PSA A* and PSA B*.

Relationship between PSA A* and PSA C*

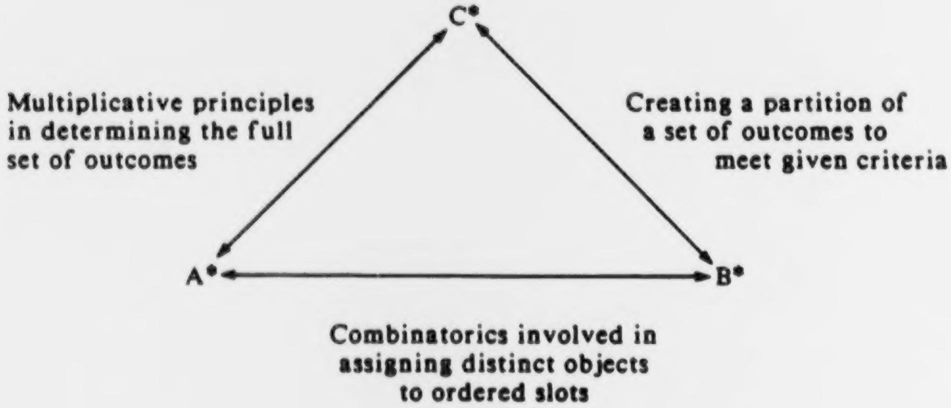
PSA A* shares with PSA C* the multiplicative structure involved in determining a possible set of outcomes. For PSA C* we must cross the number of outcomes on one probability device (coin, spinner, or die) with those of a second probability device and then again with those of the third device. For example, one might say "This die could come up either a 5 or an 8; for each of those possibilities the other die could be a 2 or a 3 or a 4; and then for each of those the coin could be plus or times." Similarly, we can think of PSA A* as involving making a choice for the first slot, followed by a choice for the second, and so on.

Relationship between PSA B* and PSA C*

PSA B* shares with PSA C* the idea of creating a partition. In PSA C* we can partition the possible outcomes into two subsets so that the two players are equally likely to win each chip. In PSA B* we must partition the set of cards into three subsets meeting the various further conditions of the problem. Note that in PSA C*, if the new rules created by a child in fixing the game do not define a partition, then there might be rolls or spins for which neither player gets a chip (e.g., if new spinner numbers create a product that is not on the board) or plays in which both players are entitled to a chip (both of which actually happened). The only arrangement of cards in PSA B* that fails to be a partition arises when some card isn't assigned to a table or shelf (which happened fairly often) because a card could not be assigned to more than one (since the cards were physically indivisible).

Finally, there is a connection between PSAs A* and B* -- the combinatorics involved in assigning objects to an ordered set of positions. In PSA A* we have four distinct objects (colors or performers) that are to be put into order. The conditions of PSA B* are arranged so that any correct solution involves three distinct numbers (e.g., 5, 8 and 10 people at the tables); given a legitimate set of three numbers, a solution can be obtained through any way of assigning those three numbers to an ordered set of three slots, where here the slots are "old, middle, and young" or "expensive, middle priced and least expensive". In B*, of course, finding all these solutions was not part of the problem (and in fact nobody recognized that there were many ways of doing it), but the principle is exactly the same as in the PSA A*. Another way to think of this connection is to imagine that the three clock shelves are to be painted three different colors, or that three of the circus performers are hired to wait on the party tables.

The figure below summarizes the interrelationships among the three PSAs.


C*
Multiplicative principles in determining the full set of outcomes
Creating a partition of a set of outcomes to meet given criteria
A*
B*
Combinatorics involved in assigning distinct objects to ordered slots

APPENDIX II.F:

Examples of PSA Interviews

EXAMPLES OF PSA INTERVIEWS

This appendix consists of a number of verbatim protocols from the Problem-Solving Activity interviews. The sample consists of three PSA A' interviews, two from PSA B', and two from PSA C'. (We have chosen to restrict the examples to the prime set (i.e. PSAs A', B' and C') because the aim is to illustrate differences in children's approaches to the same problems.) We present these samples in the same order as the problems were administered: C', B', and A'.

The set given here includes boys and girls from viewing and nonviewing schools, with a variety of interviewers, and from both the pretest and the posttest. No child appears in more than one interview. The seven interviews in this Appendix, together with the example of PSA C' in Chapter 4, are provided simply to suggest a range of approaches to these problems that children commonly take.

Along with each interview there is a brief description of how points for various problem-solving actions and heuristics were assigned. (These points, in total, form the P-score.) At the end of each interview is an explanation of how the M-score for the interview was assigned. The reader is encouraged to consult with Appendices II.G and II.H for details about assigning P-scores and M-scores, respectively.

Student ID 85
PSA C'
DOC: 4240M

[The introduction to PSA C', which begins this Day 1 interview, is not included here. It is similar to the introduction that appears in the excerpt from a PSA C' interview at the end of Chapter 4.]

R: . . . And when you think you've found out what's wrong with the game let me know and I'll come back and we'll talk about what you think.

S: Okay.

R: All right? Questions?

S: Mm mm.

R: No? Okay.

IIB3 (Gather data) 2pts. Gathers information by examining equipment.

(S WORKS INDEPENDENTLY FOR 3:14 MINUTES.
MOVES NUMBER BOARD SO HE CAN SEE IT MORE EASILY.
EXAMINES EQUIPMENT.
MOVES SPINNERS (DOES NOT SPIN THEM).
LOOKS BACK AND FORTH BETWEEN SPINNERS AND NUMBER BOARD.
MOVES POINTER SO THAT NUMBERS UNDERNEATH ARE REVEALED.
INDICATES THAT HE IS FINISHED.

R: Ready? Okay. . . Okay, what do you think is wrong with the game?

IIC3x (Patterns) 2pts. Pattern of results when coin is plus.

IIB5 (Consider probabilities) 2pts. Using probability.

S: Oh well um, when this (TURNS COIN OVER) lands on the plus and, you can have any number on the board and none of 'em will go to sixteen or to thirty, so the um orange has a better chance, 'cause less of the time when it lands on the plus and they all get at that green, I mean to the orange.

R: Okay, say that last part one more time. When they all land on plus they go to?

S: The orange --

R: -- Orange. And so who has a better chance?

S: The orange.

R: ...All right, um...how do you know that; how did you think about that?

IIC2 (Transform) 2pts. Transforms problem by keeping coin fixed.

S: Well you just, I just turned it to the plus sign, I just sort of added em all up and then I went to the orange. [The green and the orange the green.]

R: Mm hm. So when I left what did you start doing first?

No calculation (IIB4x) is explicitly given.

S: I just started adding and multiplying.

R: Mm hm, adding and multiplying? Okay, okay. Did you do anything else?

IIB4x (Calculate) 1pt. Counts outcomes, but not fully explicated.

S: No, this, I was adding and multiplying, I'd say it looked like the numbers up here. And most of 'em went to the um orange.

R: Okay. Was there anything else you were thinking about?

S: Mm mm.

R: Okay. I'm wondering. One person who worked on the game thought at first that it might have something to do with the weight of the coin. He thought, you know, maybe it has something to do with how much the coin weighs. And then he thought, "No, it probably isn't, isn't a good out -- idea." He decided that his idea wasn't very helpful. Did you have any ideas that you later decided weren't very helpful?

IIC3y (Pert. vs extraneous) 2pts. Repetition of the spinner numbers is irrelevant. S apparently thinks that one set of 3,4,5 on the spinner could be replaced by larger numbers (thus changing the sums and products that could be obtained); and hence that those numbers are extraneous.

S: Mmnnn, um, well there's the same numbers on this, (POINTS TO THE SPINNER WITH 3, 4, 5). That's not very helpful.

R: Say that again. There's, the --

S: -- They're all the same numbers. See the five, five, four four, three three (POINTS). They, they can have other numbers on there too.

R: Okay why, you said it wasn't very helpful to have the same numbers? Why isn't it very helpful?

S: Cause um they can just have higher numbers and when it ends on -- when it lands on plus, they can come through sixteen to thirty. So that wasn't very helpful.

R: Okay. Okay. Was there anything else you were thinking about?

S: Mm mm.

R: Okay, then what I'd like you to do next is that you told me what you think is wrong with the game and now Dr. Game wants to hire you to fix the game. So again I'm going to go over there while you're working on this and if you need anything or have any questions, just ask. Uh take as much time as you want. When you think you've come up with a way to fix the game, let me know and I'll come back and we'll talk about what you've done. This stuff is here for you to work with if you want to and if you don't that's fine too. Okay?

S: Okay.

R: Okay.

(S WORKS INDEPENDENTLY FOR 0:50 MINUTES. . STARES AT NUMBER BOARD AND THE SPINNERS. RAISES HAND.

S: Okay.

R: You ready?

S: Yeah.

R: Okay...So did you fix the game?

S: Well I think.

R: You think? What do you think?

IIB4y (Manipulate) 2pts. Describes changes to the game.

S: Well first I think that you should put, that I should put different numbers on the game (POINTS TO SPINNER) so that it won't just be the same. And different numbers over here [on the number board]. And then you can mix these numbers up and put some numbers up in orange and some numbers up in green so um they can both have an equal chance.

R: Okay. Um, I have a bunch of questions, I'm figuring out how to ask them. Let me ask about this first. You said you want to have different numbers here and different numbers here? Will you say again why you want to have different numbers.

S: So um, so some numbers can be higher so loo -- they can come over here [in Green loop] and some of em will be low so they can come over here [in Orange loop]. Then they'll both have an equal chance of winning.

R: Okay, um, did you have some numbers in mind that you would put on the spinners?

S: Uh, I'm like um, probably uh like eight and three and two.

R: Okay, like both spinners or at one spinner?

S: Um, I guess I'd just put em on this spinner [the one with 3, 4, 5].

R: Put more numbers on this spinner? Okay. Does it make a difference which numbers or just more numbers.

S: Just higher and lower numbers.

R: Higher and lower numbers? Okay. Why higher and lower?

S: Cause the higher numbers, well mo -- well most of em will come to, to sixteen and thirty. And the lower will come to one and fifteen.

R: Mm hm, okay. And then did you want to do something with this one [the other spinner]?

S: Um uh.

R: Or no?

S: No.

R: No? Okay. Okay and then you said you wanted to do something with this [the board] also. Can you tell me again what you wanted to do there?

S: Well you can um mix these numbers up and put, put em up here and um get some of these numbers and put em over here.

R: So some of the numbers from the green loop could go on the orange loop, in the orange loop, and why would you do that?

S: Cause uh, so this one um only have lower numbers and this one only have higher numbers so they can just both get the numbers.

R: Mm hm. And why do you think that that would fix the game?

S: Well because uh they're high. Well sometimes they're all, they're all high [the numbers now in Green] and they're all low [the numbers in Orange] so you can just have some over here and they can be um high numbers. And they [high numbers] can be up here [in Orange] or they can be low numbers and they can be up here, down there [in Green].

R: Mm hm, mm hm, okay. I thought I heard, let me ask it and make sure that this is what I heard. I thought you said something before about having an even chance. Is that what you said? I'm not sure if I heard you right.

S: Yeah but um I was saying that they would have an even chance if they changed these numbers [on the spinners].

R: Okay but that they would, these guys would have an even chance--

S: --Mm hm--

R: --or, okay. If you changed these numbers. All right. Anything else you were thinking about?

S: No mm mm no.

R: Okay. Let me ask, when I left and went over there and you started thinking about this, what did you think about first?

S: I just turned over here and I was thinking about these numbers [on the number board]. And I was saying, "Well maybe you can change them and just move em around."

R: Mm hm.

This is apparently all part of the same solution, not alternatives.

S: Well and then, then I said, "Well yeah." And I looked over here [spinners] and I said, "Well we can change these around too."

R: Change the spinner numbers around too? Okay. When you were working on this, did you think of anything that you later decided wasn't a very helpful idea?

S: Mmm, just, these numbers here and these numbers here, so they can just both be mixed and they can have it even. So the numbers right now I don't they're very helpful.

R: The numbers where they are aren't very helpful? Okay. When you're thinking about what you can do to fix it, did you have any ideas about fixing it that you decided weren't very helpful?

S: Nnno, mm mm.

R: No? Okay. Anything else you were thinking about with this?

S: Mm mm.

R: No?

S: No.

R: Okay. Then. Let's go on to the other one.

[The interview continues with PSA B'.]

• • •

The P-score for this interview is 13 (6 actions or heuristics of 2 points each, plus 1 with 1 point).

The M-score is 10, from the following subscores:

E (4 points) for a statement that Orange is favored;

H (2 points) for a more general statement about how the game favors Orange;

O (3 points) for one correct statement of the form "Orange wins every time [something] happens," where the adverbial clause comprises two or more combinations; and

D' (1 point) for suggesting a change in the arrangement of numbers in the Green and Orange loops to make the distribution of the obtainable numbers more evenly split between the two loops. (No further points are given here because there is no explicit consideration of exactly how moving the "high" and "low" numbers would affect the game.)

Student ID 79
PSA C'
DOC: 4129M

[The introduction to PSA C', which begins this Day 1 interview, is not included here. It is similar to the introduction that appears in the first excerpt from a PSA C' interview.]

R: (INTRODUCES PSA C', THE SPINNER GAME.) When you think you're found out what's wrong with the game, let me know and I'll come back and then we'll talk about it and I'll ask you some questions about what you were thinking about.

S: Okay.

R: Okay? Any questions? No?

S: Oh yeah, I have one question.

R: Okay.

IIB3 (Gather data) 2 pts. Gathers information by asking question that has not already been answered.

S: Did um, did he have one game like, he had one before like another set before? Like another game before he had this one broken into?

R: Um --

S: -- does it, does he have a right one, the correct kind?

R: No, he had the factory where he had all of his games, and Mr. Enemy broke in and changed all of his games. So, but Dr. Game, who invented them, knows that there's something wrong with it and you're hired to find out what's wrong. Okay? Does that answer your question?

S: Mm hmm.

R: Okay. Any other questions?

S: Hn hn. (SHAKES HEAD NO)

R: No? Okay.

(S WORKS INDEPENDENTLY FOR 2:29 MINUTES.

IIB3 (Gather data)
2 pts. Examines pieces.

PICKS UP GAMES PIECES; LOOKS AT BOTTOMS OF SPINNERS, PLAYERS.
WIGGLES (DOES NOT SPIN) POINTERS.
TAPS BASES OF PLAYERS.
LOOKS AT BACKS OF PLAYERS, BACKS OF PLAYERS' SIGNS.
REMOVES PLAYERS FROM THEIR STANDS, REPLACES.
LOOKS AT PLAYERS, COMPARING.

IID2 (Alt. solutions) 2 pts. This is an alternative solution.

No more points for IID2 here, since this is the same kind of alternative solution.

S: Okay, I found something wrong with it, two things... they, one thing, this, this one [Green player] is bigger than this one [Orange player], and this one (POINTS TO THE BACK OF GREEN'S SIGN) is a, like a cardboard and this one (POINTS TO BACK OF ORANGE'S SIGN) is like a, like a, in-index card and um, let's see, there's one more thing. This guy's arms are fatter than the other, see the little slits, see how this one's bigger and this one's smaller. And that's about it.

R: Why does that make the game wrong?

S: Well, for one thing...I don't know...uh...well you probably couldn't sell it that good it or something because it's, it would look kind of bulky and so on and so forth. And stuff like that.

R: Okay.

S: That's about it.

R: Okay...Is there anything else you were thinking about?

S: No.

R: Can you tell me what you did first? I mean, I, I told you about this and then I went down and sat down. What did you do? Or what were you thinking about?

S: Just thinking that, I hope I find what was wrong with, with the game.

R: Mm hmm.

S: That's it.

R: Okay. What did you do?

S: First I um,...uh...I looked at these, (POINTS TO SPINNERS) then I went and I looked at this (POINTS TO BOARD) and the last I looked at the dice.

R: Okay. Let me ask you, you said you looked at the spinners?

S: Mm hmm.

R: What did you think when you were looking at the spinners?

S: That um, these, these right here, (TOUCHES SPINNER POINTER) could of been made out of, like um, cardboard or something instead of wood. And the, and the, the base, (TOUCHES SPINNER BASE) this right here, could of been made out of cardboard, too, like regular games.

R: Okay, then, and what did you think about that?

S: What do you mean?

R: Well, you said that if they could have been made out of something different.

S: Mm hmm.

R: Does that make it, does that make it wrong, that they were made out of, well what did you decide after you were thinking that?

S: What did I decide? Um...I don't know...Let's see, um... I really don't know.

R: Okay. Can I, let me ask a different question. Would it make a difference if they were made out of something else?

S: Yeah, they would be different.

R: Would it make a difference in the game if they were made out of something else?

S: Well yeah because they would be cheaper and um, would be cheaper on, on the guy who bought it and then, see the wood'd be more expensive 'cause just buy like one sheet of card-cardboard or poster board and then would be um, would be better.

R: Okay.

S: And and then, another difference is like, the people who buying it, it [the pointer] would probably bend and stuff and then this would probably break off or something.

R: Okay. Okay. You said you also looked at the, the, at this thing? (POINTS TO NUMBER BOARD.)

S: Mm hmm.

R: What were you thinking about, with that?

S: Uh, I get um...I thought that he would be like, he was the enemy guy, he would um, he wouldn't cause this, this stuff to come off the paint, (POINTS TO SURFACE OF BOARD) to come off this but then I looked at it again and how it has this nail and how it has a rough surface, they just come off like that.

R: Mm hmm. Okay. Is there anything else you were thinking about?

S: Mm mm. And then I went on to the guys and that's it.

R: Okay...Let me ask you one more thing here. One person who worked on the game, thought at first that the problem might have something to do with the weight of the coin, how much the coin weighs. And then he said to himself, no, I don't think that, no that's not going to be very helpful. I'm wondering if there's anything that you were thinking about where you decided no, anything else where you decided, nope that's not a problem, but it was something you thought about?

S: No, I wasn't thinking about the coin.

R: Or anything else in the game. I mean, I use the coin as an example.

S: I was just, just used, thinking about these, these guys, and this board. That was it.

R: Okay.

S: And, I didn't, I didn't bother to look at the coin or these chips.

R: Is there any reason?

IIC3y (Pert. vs extraneous) 2 pts. Chips not relevant.

S: Well, they looked, they looked fine to me. Nothing wrong with them. Plus this coin um...um. Guy just lost twenty-five cents, that way.

R: Because?

S: Cause, he, he glued um, the plus and times on this, so he lost twenty-five cents. He could of bought, he could of bought five pieces of gum with that.

R: Okay.

S: And um, that's it.

R: Okay. Um, so let me go back again, so what do you think is wrong with the game?

S: Let's see, I said that, this guy's arms were fatter than the other and um, and then, this was, was bigger than this, and, and as you see here, [backs of players' signs] see how, made of a different thing than this. And that's it.

R: Okay.

S: So the enemy guy must've like, he could've, he could've like changed this around, (TOUCHES SIGNS ON PLAYERS) and then he could've, he could've made the coin like this, and probably there was another coin.

R: What do you mean, he could've made the coin like this, probably there was another coin?

S: See like, he could've he could've had coin in his pocket, uh, he could've had a quarter in his pocket and then just glued plus and times to make it, make it look, make it look like a real coin. And then um, and then there was probably another better coin than this to go with it.

R: Like, what would be a better coin?

S: Well like um, a layered gold coin, something real fancy and stuff.

R decides to get ideas on how to fix without giving more time to work alone. Not much is happening, and R knew from previous discussion that S was eager to leave because he was missing Physical Education.

R: Okay. Okay. Let me ask you something else. Dr. Game was interested in hiring you to find out what's wrong with the game. And now he wants you to fix it. Okay? So, what I'd like you to do is to think about what would have to be done to fix the game. Could you talk a little bit about that?

S: Well first he would get a new coin that was dipped in gold. And then, and then um, he could like fix this [signs] to get the right texture.

IIB4y (Manipulate) 2 pts. Describes changes to the game.

(TOUCHES TAGS ON PLAYERS.) And then, and then, he could get the right brand and stuff. And then he could um make this guy's arms better, cause, see how they're like that? See it's little and this one's big? And this guy is narrow and this is large?

R: Mm hmm.

S: And then he could like get something, something better to, to um, so the paint won't rub off (TOUCHES BOARD) and that'll will make the game look kind of funny. And then he could sell it better.

R: I, I have another question. Why should he do all of this, why would this fix the game?

S: Well, it would give him more profit and stuff, people would start buying it. And then he would have a lot of money so he'd make more in it. He has profit he could do what he has to do with his bills and stuff.

R: Okay. Is there anything else you were thinking about with this game?

S: Mm mm. Just, just, just what I said.

R: Okay.

S: That was it.

R: Okay . . .

[Interview continues with PSA B'.]

• • •

The P-score for this interview is 10 (5 actions or heuristics, each with 2 points).

The M-score is 1, from "B. Any physical feature of the equipment . . . if the reason why is that it's different from, or doesn't conform to, S's idea of what spinners, coins, etc., should be like or were in fact like originally."

Student ID 24
PSA B'
DOC: 4139M

R: Let me get rid of this [equipment from PSA C'] and show you something else ... Before I start this one, I gotta make sure that you know what "twice as many" means. Two times as many? You know, two times as many or double? For example, what's twice as many as ten?

S: Twenty.

R: Right. And what's twice as many as twenty?

S: Forty.

R: Okay. You make up an example. Think of some number, some small number, and then tell me twice as much is?

S: Um, what is twice as five?

R: Five okay. And what is twice as much as five?

S: Ten.

R: Ten. All right, so you know that. Now one other thing, I wanted to know if you know the word "optional"? What does the word "optional" mean? You know what "optional" means?

S: No.

R: Okay I'll tell you. When something is optional, it means that you don't have to do it. You can do it if you want to, but it's up to you, it's your choice. You have a choice in other words, for an optional thing. Now, let's pretend that you run a store that sells different kinds of high quality clocks. Now I don't have the clocks with me, but I do have some price tags. These are price tags of clocks. Like this (SHOWS TAG) is a clock made by the company, Offbeat Incorporated, and it costs forty-eight dollars. And here's a clock (SHOWS TAG) made by the Tempest Company, and it costs sixty-one dollars. All right? So these are different price tags for clocks. Some clocks have the same price, but most of them don't. Most of them have got different prices on them. Now what you want to do is set up a special display case for these clocks. And the display case has three shelves. You want there to be a shelf for the more expensive clocks, a shelf for clocks with prices in the middle, and a shelf for the less expensive clocks. Every clock on the more expensive shelf has got to cost more than every other clock on the other shelves. And every clock on the less expensive shelf has got to cost less than all the other clocks on the other shelves. All right? Also, there should be at least four clocks on each shelf. You can have more than four clocks if you want to on a shelf. But there's got to be at least four. And there's one other thing, and this is optional, you don't have to do this, but if you want to, you can try to make one of the three shelves have exactly twice as many clocks, that's two times as many clocks, as one of the other shelves. And the third shelf can have whatever is left. Now that was sort of complicated, so I've got this thing here (SETS UP THE INSTRUCTION CARD THAT SUMMARIZES THE CONDITIONS OF THE PROBLEM). I'll repeat it and give you this. (POINTS TO INSTRUCTIONS WHILE READING THEM.) Assign all the clocks to three shelves by making three piles of price tags. Make a shelf with more expensive clocks, one for the middle priced clocks, and one for the less expensive clocks. Each shelf must have at least four clocks. That means four or more clocks. Right? And this optional thing: make one of the three shelves have exactly twice as many clocks as one of the other shelves. Remember this last part is optional. When you've assigned each clock to a shelf by making one pile of price tags for each shelf, let me know and we can talk about what you've done. Okay? Does this make sense? Have any questions?

S: No.

R: I'm going to go over there while you're working on this, and if you need anything or need to ask me any questions let me know and, oh don't forget there's a bunch of stuff over there (POINTS TO EQUIPMENT KIT) to use if you, if you want to. And when you're finished say, hey [name of interviewer], and I'll come back and we'll talk about what you've finished. Oh, I may have to interrupt you before you're finished, but that's okay, and at that point we'll talk about what you've been doing and what you've been thinking. And as I said before, I'm interested in how you think about these things. So there you go, there's your price tags and I'll be right over here.

(S WORKS INDEPENDENTLY FOR 1:45 MINUTES.
SORTS THE CARDS ONE BY ONE INTO 5 PILES.

IIB4y (Manipulate) 2pts. Manipulates cards.

LOOKS THROUGH EACH PILE.
SPREADS OUT SOME OF THE PILES, FORMING 2 ROWS OF 4 CARDS EACH.

IIB3 (Gather data) 2 pts. Gathers information by looking through all the cards.

S: Um [name of interviewer]?

R: Yeah?

IIB3 (Gather data) 1pt. Gathers information via asking question that has already been answered.

S: Could these (INDICATES NUMBERS ON TWO TAGS) be the same?

R: Yeah, some of the prices are the same.

S: Okay.

R: Most of them are different, but some of them are the same.

This is all a continuation of the manipulation of cards, IIB4y.

(S CONTINUES TO WORK INDEPENDENTLY, FOR 2:10 MINUTES.
INSERTS A THIRD ROW OF 4 CARDS BETWEEN EXISTING ROWS.
SPREADS OUT ALL CARDS, NO LONGER IN ORIGINAL PILES.
INSERTS 2 CARDS IN MIDDLE ROW, GETTING 3 ROWS, OF 4, 6, 4 CARDS, RESPECTIVELY.
COLLECTS REMAINING CARDS INTO A SINGLE PILE.

S: [Name of interviewer], I'm finished.

R: Finished ... How did it go?

S: Well I think these (POINTS TO TOP ROW) are the highest and these (MIDDLE ROW) are the middle priced and this (BOTTOM ROW) the cheapest.

R: Okay. Um ... Okay let's see. Tell me that again; I'm just looking at what you're doing here.

S: Okay. These (POINTING AT THE THREE PILES IN TURN) are the highest, and these are the middle priced and there's the less, the less expensive.

R: Okay. All right. Um, and you've got them spread out here. So let me just check this (INDICATES CARD SUMMARIZING CONDITIONS) over: Assign all the clocks to three shelves by making three piles of price tags. Did you do that?

S: No. (STARTS TO MOVE CARDS IN THE FOURTH PILE.)

R: Uh ...

(S SPREADS OUT THE REMAINDER CARDS, AND INSERTS THEN AMONG THE EXISTING ROWS, GETTING ROWS OF 6 CARDS ($28-31), 13 CARDS ($35-52), AND 4 CARDS ($53-67).

R: Okay? (READS FROM SUMMARY CARD.) Assign all the clocks to three shelves by making three piles of price tags. And that's what you've done right? (S NODS) You've got all of them out there. (READING FROM CARD) Make a shelf for more expensive clocks, one for middle priced clocks and one for less expensive clocks. Okay, which is the more expensive?

S: This one (POINTING).

R: Okay. And how do you know that um, that they're all more expensive than all the other clocks?

S: Well cause, these are the highest numbers with the price tags.

R: Okay. And which is your middle priced shelf?

S: All of these (POINTING).

R: Alrighty. And your less expensive?

S: These (POINTING).

R: Okay. And um, let me see. And how do you know that these (POINTING TO THE BOTTOM ROW) are all less expensive than everything else?

S: Cause they're the less, the less expensive, the lowest, they cost the lowest.

R: Okay. All right. Um, now each shelf must have at least four clocks. Right? Have you got that?

S: Yes (POINTS).

R: All right. And did you try the optional?

S: Um, no.

R: Okay. Why, I know I said it was optional, is there a reason you didn't try it, or...?

S: Uh, 'cause...

[S indicates that she did not really try the optional part. She is not sure if it is possible to do the optional part or not.]

R: ...How did you do it first? What was the first thing that you did when you started working on this?

IIC2 (Transform) 2pts. Transforms problem by organizing cards by decades.

S: Well I separated them to twenties in one pile and then thirties in the other and the forties and the fifties and the sixties.

R: Oh, okay. And then what did you do?

S: And then I got um, the sixties and put them on the highest and then I got like the fifty-sevens and like that. On the middle ones I got like some of the fifties and forties, all the forties, and about three, three of the thirties. And the lowest, I got the lowest numbers that that would be. And that's all I done.

R: And so this is just the way it worked out or did you, did you do anything else, did you move them around any or...?

S: Um, no it's just the way I did.

R: Okay. So this is the way it was? (S NODS.) Okay. Well that's great. Uh, that's all the questions I have for you, do you have any questions for me?

S: Um...no.

R: Okay. I really do want to thank you for helping us out. It really is very very helpful to us... [Discussion of where the next day's interview will take place.]

* * *

The P-score for this interview is 7 (3 actions or heuristics with 2 points each, and 1 with 1 point).

The M-score is 7; this results from the following subscores (see the M-Score Coding Guide, Appendix II.H, for details):

B (1 point) for having the correct number of piles;

D (1 point) for having at least 4 cards in each pile;

H (3 points) for having the piles with correct order relationships among piles;

K (2 points) for justifying that order[1]; and

L (0 points) for not attempting the optional part.

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[1] Most children find it very difficult to answer the question about how one knows that all the clocks on the most expensive shelf are in fact more expensive than all the other clocks. Clearly they are not making dozens of pairwise comparisons, comparing each clock on the most expensive shelf with each of the other clocks. Instead, they are using some implicit form of transitivity -- e.g. the cheapest clock on the most expensive shelf is more expensive than every clock on either of the other two shelves. The child who simply says something like "These are the highest numbers" is awarded the full 2 points for justifying the order.

Student ID 42
PSA B'
DOC: 4204M

[The first part of this Day 1 interview, dealing with PSA C', is not included here.]

R: (INTRODUCES THE CLOCKS-ON-SHELVES PROBLEM, JUST AS IN THE PREVIOUS EXAMPLE, INCLUDING THE INTRODUCTORY REMARKS ABOUT "TWICE AS MANY" AND "OPTIONAL.")

(S WORKS INDEPENDENTLY FOR 6:22 MINUTES.

IIB3 (Gather data) 2pts. Gathers information by examining all cards.

SPREADS OUT ALL THE CARDS RANDOMLY OVER TABLE.

MOVES THE CARDS INTO 4 COLUMNS.

IIB4y (Manipulate) 2pts. Manipulates cards.

SEARCHES THROUGH CARDS, SELECTING ONE BY ONE, AND FORMS 1 ROW OF 5 CARDS ACROSS TOP OF TABLE.
STARTS SECOND ROW; ADDS ONE CARD TO TOP ROW.

IIB4x (Calculate) 2pts. Counts cards.

COUNTS CARDS IN ROWS; THEN MOVES CARDS SO THERE ARE 8 IN THE TOP ROW.

IIB6 (Trial & err; guess & chk) 2pts. Trial and error adjusting.

RECOUNTS ROWS, ADJUSTS BY SHIFTING 2 CARDS FROM MIDDLE ROW TO TOP ROW (WHICH NOW HAS 10).
ADDS CARDS TO MIDDLE ROW UNTIL THERE ARE 8.
PUTS REMAINING CARDS IN BOTTOM ROW.
THE FINAL ARRANGEMENT HAS 10, 8 AND 5 CARDS, FROM TOP TO BOTTOM, AND THE 3 ROWS ARE CENTERED ON THE TABLE.

S: Finished.

R: Finished? Okay. Well okay, so can you tell me what you did? Did you get --

IIB4x (Calculate) 2pts. Doubling calculation. (Note that this is a different use of IIB4x from the one above.)

S: -- Well, kinda make it look kind of fancy because store's usually want to have things fancy so one of -- the top row has more, more than, than this shelf (POINTS TO MIDDLE ROW). And then this shelf [middle] has more than this [bottom] shelf. And they're all in order. And this one (POINTS TO TOP ROW) has eight -- no, this one has ten -- this one (POINTS TO BOTTOM ROW) has five; it's double.

R: Okay. All right; so I was over there when you were working on this. How did you start? What did you do? --

Like what did you do first?

S: Well, first I um, I put em all up, I mean in row [the cards were not in any rows initially] to, so I could look at them all without having them all on top of each other. And then I took out the largest one and looked around again to make sure there wasn't any more of that, of that or higher. So I took that, put that one up there [at top of table], and I kept doing that till, I kept switching them ar -- switching like some of these to over here, to make it even and stuff in here.

R: Now tell me, I don't understand what you mean by, you said you were shifting some or switching --

S: -- Or first I started out with, with six cause I was gonna do one row with three. And it says has to have more, at least four, so I was gonna do eight, so I decided to do ten and I, I just kept taking these over there to make it more even.

R: Okay when you say "these," what do you mean? "Kept taking these over there," that's what I don't understand.

S: 'Cause like whenever, I started out with six, (COUNTS ALONG TOP ROW, FROM LEFT TO RIGHT) one two three four five six, and these [remaining 4 cards] were, were going to be in this [middle] row. And it's the ones to make, I took the next highest one from this [middle] row and put it right there (POINTS TO TOP ROW).

R: Okay. I'm not sure I follow you.

S: ...Yeah...These [rightmost 4 cards in top row] weren't in yet. Neither were these [in middle and bottom rows]. (MOVES ALL THE CARDS TOWARD THE BOTTOM OF THE TABLE EXCEPT THE LEFTMOST 6 IN THE TOP ROW.) I had it like this, at first.

R: Okay.

IIC4 (Reapproach) 2pts. Reapproach problem by rejecting first idea of using 6 cards on top row.

S: And then I, and then I figured out that couldn't do, cause I wanted to do the optional. And so I decided I couldn't, I couldn't do three in one shelf because it ha -- it said you have to have four. And so I took the next highest one from this [middle] row and I stuck it, put it there [top]. And I did it again until I got, got up to ten.

R: Okay.

S: Ten. Then I did this row with eight. (RECONSTRUCTS MIDDLE ROW OF 8.) See, and these [cards at bottom] were still over here and I had to... (MOVES CARDS.) Then I did this row with five...

R: Okay now what, I guess what I don't understand... I think I follow everything that you said, up to the point where you said you shifted, you started out with six here [top row] you said. All right, and then you said that that wouldn't work. Then you put ten in this top row. How did you decide, why did you do that? Why ten?

S: Because the bottom I was gonna have um five and if I had four then there'd be another one here [middle] and then there'd be ten [top], nine [middle] and then it'd skip way down to four.

R: Uh huh.

S: So it wouldn't look so plain on the bottom, like there's not very many of those left 'n' stuff. So I tried to get as many as I could on the bottom with having the double up there.

R: Oh I see, I see. So you, which did you decide on first, this, the five here [bottom] or the ten up here [top]?

S: The ten up there.

R: Okay. All right. And how do you know that every clock on this top shelf is more expensive than every other clock?

S: Because 63 is less than 67 [LAUGH] and all the way down.

R: Okay, all right. And I guess you know that every clock on the bottom is less expensive than all the other clo --

S: -- Cause then 28 (PICKS UP CARD WITH $28) wouldn't go up here [in top row] in between 57 and 53.

R: Right. Okay. And so you did decide -- why did you decide to do the optional part of it?

S: It would give me something [LAUGH] it's why if I was doing this at home or something it would be ... it would ... Um, like if I was playing this game at home and I didn't have anything else to do, then it would keep me busy.

R: Hmm. Okay.

S: Switching 'em around and stuff.

R: Mm hm. All right. Now, when I was talking with somebody before in New York she said that the first thing she thought of doing was to put em all in alphabetical order. She thought like putting this Acutime in with a Clockworks (POINTS TO CARDS) and so on and. And then she realizd, no, that doesn't make any sense, that wouldn't work.

S: That's not the way, that's not, they're not priced by the alphabetical order.

R: Right.

S: It's a type of clock.

R: Yeah, well, that's what she realized. And she said, "I thought about doing that but then I decided that wasn't helpful." Did you think about anything like that that you later decided wasn't helpful?

S: Well with these two (POINTS TO TWO CARDS ON BOTTOM ROW) I just, I was just gonna stick em in there, I said, well let me get, er [not clear; sounds like "or touchy put 'em in put those two a P and W"].

This is not coded for IIC3y (Pert. vs extraneous) or IIC4 (Reapproach).

R: Say it again, put what?

S: With these two, um P comes before W, I was just gonna stick 'em any -- like, not anywhere, but I mean in any order in this [bottom row].

R: Oh I see, I see. But you decided to put the P before?

S: W.

R: Before W. Okay. Okay anything else?

S: No.

R: Okay. Well, [name of S], this has been very helpful. That's all the questions I have for you. Do you have any questions for me?

S: No.

R: Okay. Thanks really very much for helping us, 'cause it really is very helpful. [Name of Day 2 interviewer] will be here tomorrow.

S: Okay.

* * *

The P-score for this interview is 12 (6 actions or heuristics with 2 points for each).

The M-score is 12 (the maximum M-score for this PSA); this results from the following subscores:

B (1 point) for having 3 piles;

D (1 point) for at least 4 cards in each pile;

H (3 points) for having the correct order relationships among the piles;

K (2 points) for justifying that order; and

L (5 points) for making two piles with a correct twice-as-many split, and saying that one pile has twice as many as the other.

Student ID 44
PSA A'
DOC: 4449M

[The first part of the Day 2 interview, concerning the previous day's experiences with PSA C' and PSA B', is not included here. The following excerpt starts with the introduction to PSA A', the circus performer problem.]

R: ...Okay. Now I have another thing here. This is about a circus owner who's about to take his show on the road; they're touring around the country for a while. And his circus has only four performers, and here they are. A juggler, here's the juggler, an acrobat, a magician and a clown. (DISPLAYS THE FOUR CARDS WITH PICTURES OF THE PERFORMERS AND THE "PROGRAM" WITH A ROW OF FOUR RECTANGLES INTO WHICH THE PERFORMER CARDS CAN BE PLACED.) Those are the four performers. And each person here is going to um perform every night. The circus owner wants them to perform in as many orders as possible. One order for example would be to have the juggler, the magician, the acrobat and the clown. (PLACES THOSE CARDS IN ORDER, FROM LEFT TO RIGHT ON THE "PROGRAM".) And this means that on this particular night, the juggler would be first, the magician would be second, the acrobat would be third and the clown would be fourth. Can you show me another order?

S: (SWITCHES ACROBAT AND CLOWN)

R: Okay. Uh now it's your job to find the total number of orders that the circus owner can make.

S: Okay.

R: Okay? And I'm going to go over there while you're working on this. And when you think you've come up with the largest number of orders, and you're sure that you haven't repeated any of them, let me know and I'll come back, and then we can discuss what you've done. Now I may have to interrupt you before you finish, but that's okay. And at that point we'll talk about what you've been doing and what you've been thinking. And as I said before, I'm interested in how you think about things. Oh and don't forget there's a bunch of stuff over there (POINTS TO KIT OF EQUIPMENT) to use if you want to. I'm just going to go over here.

S: Okay.

R: Okay?

(S WORKS INDEPENDENTLY FOR 9:27 MINUTES.
GETS PAPER AND PENCIL.

IIC1x (Diagram, etc.) 2pts. Lists orders.

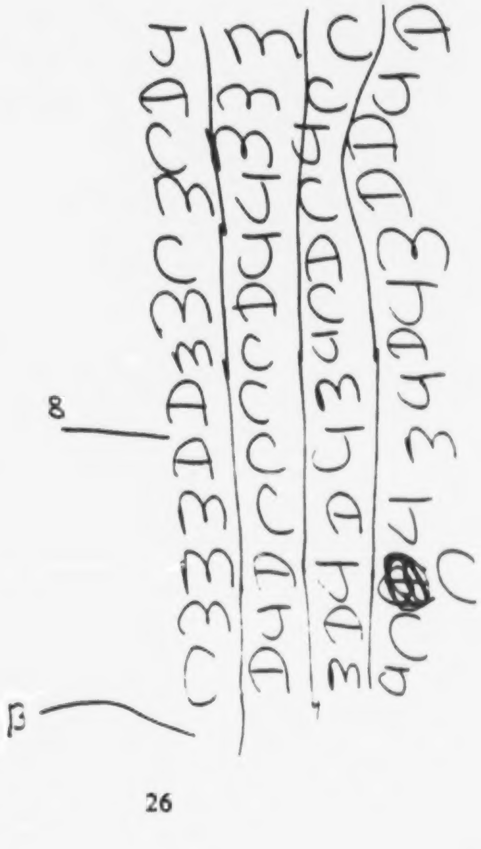
WRITES 3 ROWS OF LETTERS, WITHOUT MOVING CARDS.
STOPS WRITING, PUTS PAPER ASIDE.
STARES AT CARDS.

No IIB4x (Calculate); Can't be sure if he's calculating.

USES CALCULATOR VERY BRIEFLY.
STARTS WRITING AGAIN.
STARES AT CARDS.
TAPS EACH CARD WITH PEN, AS IF COUNTING THEM.

IIB4y (Manipulate) 2pts. Moves cards.

STARTS SWITCHING CARDS, WITH NO APPARENT SYSTEM, WITHOUT WRITING.
TAKES ALL CARDS OFF THE PROGRAM BOARD, THEN PUTS THEM BACK ON.
GETS CALCULATOR AGAIN, PUTS IT ASIDE COUNTS ORDERS OBTAINED SO FAR (8) AND WRITES "8".
CONTINUES TO WRITE.
STARES AT PAPER; LOOKS BACK AND FORTH BETWEEN CARDS AND PAPER.
AT THIS POINT HIS LIST LOOKS LIKE THIS:



R: Okay I'm gonna to have to interrupt and...

S: Okay.

R: So...How are you doing?

S: Not too good.

R: If you had more time to, to uh work on this, what would you do...what are you doing here?

S: Well see (POINTS TO NOTES) "J", "J" for juggler, "M" for magician, "C" for clown and "A" for acrobat. And I was trying to, by these (POINTS TO NOTES) I was trying to make as much as I could, but it got too confusing cause I, see like, you, you might do some over and it's hard to look all of them over.

R: Mm hm. So, so, how did, uh, how many orders did you make?

S: Uh, thirteen.

R: If you had a lot of time, how many things do you think you could've gotten?

IIB2 (Estimate, approx.) 2pts. Estimate, with rationale.

S: Maybe like thirty, if I had a lot of time. Cause it's real confusing see, cause there's too many and you have to keep looking back if you'll, if you, if you didn't make one over.

R: Uh huh. Okay. So you might be able to find about thirty?

S: Mm maybe...

R: Okay, all right. How do you know that you haven't repeated any so far?

IIC1y (Use objects; act out) 2pts. Uses partial list to generate new orders.

S: Well, I keep looking back. I might have, but see like, (POINTS TO NOTES) there's like one "C" and see if there's not too many, there's no C's here (INDICATES A COLUMN) so I started using C. 'Cause as long as there's a C and then it'd be different there, or, you know. I have to keep looking back.

R: Okay. So can you tell me um, okay, suppose you had all the time you wanted and you wrote 'em all down and you were through; in other words you, you found as many as you could. Do you think that somebody else could come in and find another one?

S: Mm probably.

R: You think so?

S: Yeah.

R: Okay. Now, I as you know, I wasn't paying any attention to what you were doing when you were working on this. Can you tell me how you started this off and what you were thinking about as you as you did it?

S: Well first I just started with the all, with these, I started off doing this. (INDICATES A ROW ON THE LIST.) And then I, when I got to eight I got, it said, I, I said to myself this is getting too comp -- compli -- complicated --

R: Hm hm.

IIC4 (Reapproach) 2pts. Uses new approach to task.

S: -- so I thought of a way uh like I can multiply it to get an answer but I couldn't find any way to like work, any way that way so I went back to this [making the list]. I kept thinking i -- a way I could multiply it to give me an answer. Like four times four.

R: Uh huh. Now why did you think that?

S: Well it'd, it'd be an easier and faster way and I could get more, I probably could get more, 'cause see you know it's too confusing and, and I've only got 13 so, that'd be real hard to do it this way. I was trying to find an, an easier way.

R: Mm hm. Okay, but you were saying something about multiplying. Can you tell me more about that, what you were thinking?

IIB4x (Calculate) 2pts.

IIC3y (Pert. vs extraneous) 2pts. Multiplication is extraneous.

S: I was try -- I was trying to like find a way like I could multiply it to, to get an answer. But I couldn't cause there's all these different ways. Four times four wouldn't work cause it's just sixteen. And there's probably more than sixteen.

R: Okay. Now why do you think there's probably more than sixteen?

IIC3x (Patterns) 2pts. Describes pattern.

S: Cause you can just like change two or like three. (POINTS TO PERFORMERS.) You can change these [three leftmost cards] all different ways (MOVES HAND IN CIRCLE) and leave that [card on right] alone, and change these [three rightmost cards] all different ways (AGAIN INDICATES CYCLE) and leave that [card on left] alone.

R: Mm hm. Okay. All right, so let me go back to this. You started with J, M, C, A [referring to the first row in the list]. So you had the juggler and the magician and then the clown and the acrobat. Then how'd you get the next one?

S: I just did different ways, I made a different sequence.

R: Mm hm. And then how 'bout the third one?

S: I did it again, the same way.

R: Okay and, and why did you draw these here? (POINTS TO THE LINES AFTER THE 8TH AND 13TH ROWS ON THE LIST)

S: To number them.

R: Oh okay, okay. So did you realize or discover anything while you were doing this?

S: Yeah that this would be--take too long and be too complicated. And I, I thought there could be an easier way.

[This PSA interview continues briefly, with S talking about how there must be an easier way to do the problem. The remainder of the Attitude Interview is also not included here.]

* * *

The P-score for this PSA is 16 (8 different actions or heuristics, each with 2 points).

The M-score is 6; this results from the following subscores (see the M-Score Coding Guide, Appendix II.F, for details):

> D (3 points) "Some systematicity as shown by the explicit use of previously generated orders to create new ones...";
>
> H (1 point) "Was interrupted and realizes that there are more to be found"; and
>
> M (2 points) "Consistent checking of new orders against the list of orders already generated."

Student ID 34
PSA A'
DOC: 4118M

[The first part of the Day 2 interview, concerning the Day 1 experiences with PSA C' and PSA B', is not reproduced here.]

R: (INTRODUCES THE CIRCUS PERFORMER PROBLEM, JUST AS IN THE FIRST EXAMPLE OF PSA A' EXCERPT.)

(S WORKS INDEPENDENTLY FOR 8:30 MINUTES.)

IIB4y (manipulate) 2pts.
Manipulates cards.

MOVES CARDS IN APPARENTLY RANDOM WAY. SOMETIMES PAIRS OF CARDS ARE SWITCHED, SOMETIMES MORE THAN 2 CARDS ARE REMOVED FROM THE "PROGRAM" SIMULTANEOUSLY, PERMUTED AND THEN RETURNED TO DIFFERENT SPOTS.

IT IS OFTEN NOT CLEAR WHAT IS TO BE CONSIDERED AS ONE OF THE ORDERINGS (AS OPPOSED TO AN INTERMEDIATE STEP TOWARD SOME ORDERING) BUT AT LEAST 20 ORDERINGS ARE RUN THROUGH, AT LEAST IN PASSING, WITH MANY REPETITIONS.

NO ATTEMPT IS MADE TO KEEP A WRITTEN RECORD OF THE ORDERINGS PRODUCED, OR THE NUMBER OF ORDERINGS.

R: I'm gonna interrupt you.

S: Okay.

R: How did you do?

This is evidently not a reapproach to the task; it is the same approach.

S: I (think) that fifteen. The first time I did eleven but I messed up, so I started over, and I got fifteen.

R: Okay. How did you -- what were you doing? I wasn't paying any attention to what was going on here. How did you do it?

S: Well, just um the second time I started with the magician and like that. Then I take this one [rightmost card] off and lay these two [middle two] down. And that would be two. And, okay. That until I got him [second from right] down to here. [This sequence of moves is not, in fact, what he did.]

R: Hm hm.

S: And that would be four.

R: Hm hm.

S: And I just kept on keeping them in order--well, not in order, but it would be mixed up but not like that.

R: Hm hm. So, what would you do after you just did this? What would you do then?

S: Then I would like get this one [leftmost] and move it over here [second from right].

R: Hm hm.

S: And I would just move 'em around. (SWITCHES RIGHTMOST AND SECOND FROM LEFT.) That would be another one.

R: Hm hm. Okay, and then what would you do from here?

S: I'll just try to remember which ones I did.

R: Hm hm.

S: And which ones I haven't.

R: Okay. So...so if you had more time to work on this, that's what you would continue to do?

S: I guess but -- that's pretty hard. (SMILES.)

R: Well, I guess -- I don't know.

S: That's something I would have gotten.

R: Oh, okay. So how many orders did you make?

There is no evidence that this is an estimate.

S: I don't know. Fifteen.

R: Fifteen, okay. Now, when you got fifteen were you through, or was there more that you were going to do if you had more time?

S: No, I stopped there because I couldn't remember what I did after the first four.

R: Oh, I see.

S: So I stopped.

R: All right, so you got fifteen, and you were finished. Now, are you sure that you didn't repeat any in those fifteen?

S: I don't think so.

R: Do you think you got them all? All the possible orders?

S: I don't know. I guess I did.

R: Do you think that somebody else could have found more different orders? More than the fifteen that you found?

S: Maybe.

R: Okay, so I guess I saw what you did. And what were you thinking as you were doing that? This -- what you described to me? What were you thinking?

S: I was thinking like...if I was to make -- I was the one who was telling 'em where to go, I would think -- tell 'em what to go and you'd have to remember if -- which ones have gone first and which ones haven't, and you have to put 'em like first or wherever.

R: Hm hm. What do you mean, you'd have to put who-- what first?

S: Like if you put that man first and second and third the acrobat and the clown last, and you would have to remember not to put them in the same order, so um the people that were watching it wouldn't um know who's gonna come -- like those three had already come. It'd be the clown last again.

R: Hm hm. Okay...

[The Attitude Interview follows this, but is not included here.]

* * *

The P-score for this interview is 2 (just for manipulating the cards).

The M-score is 4; this results from the following subscores:

> C (2 points) "Six or more different new orders, generated with no apparent system";
>
> H (1 point) for empirical justification that all the orders have been found (i.e. 15 is all that he could find);
>
> L (1 point) for empirical justification of no repeats (i.e. that the 15 orders did not include repeats that he recognized).

Student ID 93
PSA A'
DOC: 4466M

[The first part of the Day 2 interview, concerning the Day 1 experiences with PSA C' and PSA A', is not reproduced here.]

R: (INTRODUCES THE CIRCUS PERFORMER PROBLEM, JUST AS IN THE PREVIOUS EXAMPLE.)

(S WORKS INDEPENDENTLY FOR 5:07 MINUTES)

GETS PAPER AND PENCIL.

IIB4y (Manipulate) 2pts. Moves cards.

ARRANGES THE CARDS IN DIFFERENT ORDERS, WRITING DOWN EACH ONE AS A COLUMN OF LETTERS. CARDS ARE MOVED WITH NO APPARENT SYSTEM, SWITCHING PAIRS OR TRIPLES RANDOMLY.

IIC1x (Diagram, etc.) 2pts. Lists orders.

TAKES ALL THE CARDS OFF THE "PROGRAM".

WHEN SHE HAS 7 ORDERS, SHE CONSIDERS THE LIST AND THEN CROSSES OFF COLUMNS 2 THROUGH 6.

IIC4 (Reapproach) 2pts. Uses new approach.

COMPLETES A LIST OF 6 COLUMNS, EACH STARTING WITH "C". THIS IS DONE WITHOUT MOVING CARDS.

IIC2 (Transform) 2pts. Transforms by keeping one card fixed.

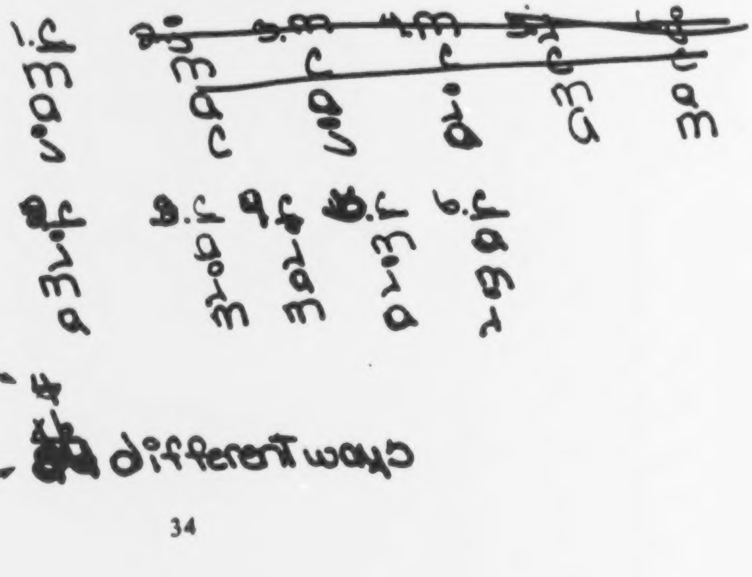
REARRANGES 3 RIGHTMOST CARDS, LEAVING CLOWN ON LEFT.

LOOKS BACK AND FORTH BETWEEN LIST AND CARDS.

IIB4x (Calculate) 2pts.

WRITES A CALCULATION (6 x 6 = 36).

AT THIS POINT HER PAPER LOOKS LIKE THIS:



[Note: This was originally "6"]

[and this was "36."]

S: I'm done now.

R: Hm hm, you're done? How many orders did you make?

S: Thirty-six.

R: Thirty-six. Are you sure that you've made the most possible orders?

S: I think.

R: You think? Okay, um, how do you think you know when you have them all?

S: 'Cause, I dunno.

R: Yeah? Okay. Um, is there a way that you have --

No more IIB4x (Calculate); this calculation has been done.

S: No! Hold it -- did I? (LOOKS AT NOTES, THEN INTO SPACE.) No, I didn't do it right. Okay. There's -- twenty-four. (SMILES AND WRITES ON PAD.)

R: Like that? Okay...so, twenty-four different orders?

S: (inaudible)

R: And are you sure that you made the most possible orders there?

S: (FINISHES CHANGING THE CALCULATION FROM "6× 6" TO "6× 4"; LOOKS UP AND LAUGHS.) I guess.

R: You guess? Okay. Um, and are you sure that you haven't repeated any of them?

S: I think so.

R: You think so? Okay. How do you know that you haven't repeated any of them?

IIB6 (Trial & err; guess & chk) 2pts. Check for duplicates.

S: 'Cause like I wrote down the -- with the clown, first, how many of them you could do, and [they're different].

R: Hm hm.

S: (?)

R: Okay...and, do you think that somebody else might be able to find other orders?

S: Not really.

R: Not really? Okay. I wasn't paying attention while you were working on this, so could you tell me what you were doing and what you were thinking? Like, what did you do first? And what were your first -- ?

S: First I was writing down all the different orders I could think of.

R: Uh huh.

S: And then I changed that because it would take too long.

This is confirmation of IIC4 (Reapproach).

R: Uh huh.

S: And I wrote down all -- like, all the different orders with the clown [first].

R: Uh huh, and you wrote down all the different orders with the clown. Okay. And then --

S: And then I multiplied by four.

R: Okay... and then what did you do?

S: (SHRUGS) Called you over.

R: Called me over? Okay. And -- just to backtrack a second, how did you know to do it that way? What were you thinking when--you said at first-you started writing it out? Is that what you said? And then you said it would take too much time? Okay, and then what was the next thing that you did?

S: I -- I got all of the orders [with clown first] and I multiplied by four. So, four times six.

R: Okay, and why did you do that?

S: Because there's four different cards.

R: Hm hm.

S: And there's six different ways you can do each card.

[Interview continues, with S explaining how one might use this information to put clocks on shelves...The remainder of the Attitude Interview is not presented here.]

* * *

The P-score for this PSA is 12 (6 different actions or heuristics, each with 2 points).

The M-score is 10, the maximum possible score for this PSA. It is derived from the following subscores:

F (5 points) for all 24 orders;

J (3 points) for a full, correct justification; and

M (2 points) for a justification that none are repeated.

APPENDIX II.G:

P-score Coding Guide

INSTRUCTIONS FOR CODING PSA BEHAVIORS
for Goal II, P-SCORE

Introduction

Sources of Evidence

Evidence to support the assignment of points to a child's behavior comes both from what he or she does and from what he or she says. The general position to take is a conservative one: do not award points unless clearly warranted by the evidence. In particular, if the visual evidence flatly contradicts a later verbal recollection, use the visual evidence.

Although the second day's interview begins with a brief discussion of what the child did on the previous day, evidence for PSAs B, B', C and C' should be taken only from the first day's interview, not from the second day.

Awarding of Points

Following this introduction, some sample behaviors are described for each of the three PSA sets (A & A'; B & B'; and C & C'). Each subgoal (e.g., IIB3) has been broken into one or more paragraphs. As a general rule, points should be awarded under any particular paragraph at most once for a given problem.

A subject who does substantially different things within the same paragraph, however, can be awarded points more than once under that category. E.g., a subject can draw a chart and get a score for IIC1x, and then draw a different kind of table or list and get additional points in the IIC1x category.

In general, though, if some behavior (e.g., adding up the numbers on a pile of cards) is repeated during the interview it should be assigned points only once, unless it is done for a qualitatively different reason, or with a different purpose in mind. E.g., if the subject adds the numbers in a pile of cards, then changes the cards in the pile, and then adds the numbers on the new cards, the subject would be given points for IIB4 only once. If, on the other hand, she adds the numbers in a pile of cards, then decides that the critical thing to look for is whether the sum is odd or even, and then re-adds another pile with the intent of determining just its parity, then she would get points for IIB4 twice.

Points are not awarded if the behavior is a response to a direct instruction from the Researcher.

How Many Points?

Each codable behavior is assigned either 1 or 2 points. (If some particular problem-solving technique or heuristic is not present, then no points are assigned.) Generally speaking, the guidelines for distinguishing between 1 point and 2 are these:

(a) Assign 1 point if there is any evidence that a particular behavior is present even at a very low level.

(b) Assign 2 points if the behavior is present and is one that:

- occupies a significant amount of time (more than one minute); or
- leads directly to, or is cited in as justification for, the subject's final conclusions about the problem; or
- is clearly identifiable as a well-formed, complete, instance of the particular heuristic of technique.

Do not award 2 points if the behavior is tangential to the main solution path (as reported by S) or occurs in passing, with little importance attached to it by S. (Note, though, that in some instances the whole point of IIC3y is to recognize that a factor is in fact unimportant; so this would get a full 2 points.)

Instructions for Coding P-SCORE for PSA A and A'

The following are some examples of behaviors in PSAs A and A' that can be coded against certain Subgoals of GOAL II:

Goals	Behaviors
IIB1 (Recall information)	Any reported recollection of information or assessment of what S has learned so far, as a result of working on the problem, so as to move forward -- e.g., "So then I asked myself how many ways I had found so far, and..."; "I wondered if I had already made that one, but I couldn't remember, so I looked at my list and found that..." (Note that the mere recall of the problem-solving steps used does not count here.) Any recollection of similar problems, whether or not they were from the pretest. (Award 2 points if S uses recalled information to solve the current problem; 1 point if the recollection pertains only to the context of the problem. E.g., "You know, this reminds me of a game that I used to play where you had to dress up Barbie in different kinds of sweaters and evening gowns, but there the sweaters were cashmere, not cardboard" (that would be a 1-point remark); or "I remembered that the last time you were here we did the same kind of thing, but then it was with stripes, so I figured the answer might be 24 again, just like last time" (this would be worth 2 points).
IIB2 (Estimate or approximate)	"I bet there are about 7 or 8 different shirts." "I've already found 14 different ways to do this, and I figure there must be about that many more that I've missed."
IIB3 (Gather info; check resources)	Asks the researcher (either immediately after instructions or by calling researcher back) for clarification of the rules, initial instructions, or other information previously presented (which should be given 1 point); or asks for clarification of something that was not already stated or implied by the researcher (could be 2 points). Do not award points for immediate confirmation -- "I'm supposed to make as many shirts as I can, right?" is confirmation (no points), but "How many shirts am I supposed to make?" is 1 point if S has already been told that that is the task, and 2 points if S has not been told. Examines stripes to see, e.g., if there is some particular way they fit together or fit on the shirt, or if they are colored on the other side; or tries to put stripes vertically, e.g., to see if they will fit.

	Looks at circus performers or the program folder to see how they fit together. (Note: Making a list of questions (as in pilot subject H.L., pretest, p. 11) is not gathering information unless the questions are actually asked.) (Note: Questions about construction of equipment, if not related to problem-solving process, do not count. E.g., "Are these pieces laminated?" does not count as IIB3.)
IIB4x (Calculates)	Calculates 4× 4 or 4× 3× 2× 1 or some other calculation. Points should not be awarded for merely counting the orders that have been created so far.
IIB4y (Manipulates)	Moves stripes or performers around in an attempt to solve the problem. [Cf. IIC1y.]
IIB5 (Consider probabilities)	"I just shuffled the stripes [or performers] around pretty much randomly, figuring that the chances were I'd get all the possible orders eventually." (Note: A statement of certainty, like "I knew that I'd get them all if I kept putting a different color in each space," (whether or not the statement is true) is not an example of IIB5, despite the interpretation that the probability is one. See IIC3.)
IIB6 (Trial & Error; Guess & Check)	(Note that making lots of different stripe orders or performer orders does not qualify as trial-and-error *per se*.) Makes a new order and then checks a list (written or in memory) of already constructed shirts or programs to see if it is a duplicate.
IIC1x (Picture; diagram; table; chart; list)	Makes a list of orders created -- i.e., a record (or just a count) of results obtained so far. Makes a list of all possible orders without moving the actual stripes or performers. Draws something like a tree diagram that shows the structure of the possible orders; draws any other kind of combinatorial diagram or chart or table.
IIC1y (Uses objects; acts out)	Uses the objects themselves (or a partial list of them) to generate new orders or to eliminate potential new orders -- e.g., "The first order started with gray and the next one started with blue, so this time I'll start with pink"; or "I looked to see what the color of the first stripe was, and since

	it was yellow I chose something different to start my next one;" or "If I try to put blue first, it's already been first;" or "I switched whatever the two people on the left were, and then the middle two people." (Any of these might also be worth some points under IIC3, patterns.) (Note that the difference between IIC1y and IIB6 is the order in which the list-checking and stripe-moving are done. If S moves stripes and then checks the list it's IIB6; if S uses the list to create new orders then its IIC1y.) Writes orders without moving stripes or performers.
IIC2 (Transform problem)	Simplifies the problem by using a smaller number of stripes or performers (i.e., 3 stripes or performers). Considers a subproblem by keeping one element fixed temporarily while moving the others around; e.g., "I'm going to keep the blue stripe on top and see what the other stripes could be;" or "Let's see how many I can make if I make the acrobat go last." If there is no verbal explanation, award points only if S keeps one element fixed and then keeps another element fixed.
IIC3x (Look for patterns)	Looks for, describes, or follows (creates, performs) patterns in color or performer changes in going from one combination to the next; e.g., "First we switch the last two colors, and then we use another color for the top stripe." (Note: Be careful not to ascribe patterns to S's actions unless unequivocal; look for verbal explanations as evidence. If there is no verbal explanation, the pattern must be repeated at least twice; note that in this set of orders the pattern is repeated only once: GPYB, GPBY, GYPB, GYBP.) Looks for, describes, or follows patterns in larger units; e.g., "If you start with the clown you get six orders, so if you start with the magician you'll also get six orders."
IIC3y (Missing information; pert. vs extraneous)	Identifies missing information, e.g., "It would depend on if all the performers really wanted to be first." "At first I thought you might want to change the shape of the neck, but then I figured that wasn't part of the problem."
IIC4 (Reapproach problem)	(Note: To get points under this heading, the S must reject a previous approach in favor of a new one, and that new approach must be pursued to the point of deciding that the new approach is or is not an effective one.) Changes from one approach to another -- (as from random to systematic or from moving objects to writing names) -- e.g., a comment like, "This

doesn't seem to be working; I wonder what would happen if I kept the pink stripe on top." (Note that the techniques used in one or both approaches might be scored elsewhere as well.)

IID1 (Reasonableness)	(Note: Don't award points here if the S is just responding to the Researcher's "Tell me what you did." I.e., mere recall of problem-solving steps is not IID1 *per se*. Similarly, don't award points here if S is responding to "How do you know?" with an explanation of what he or she did.) Makes comments like "Only 11 ways of doing it? That can't be right; there's gotta be more ways than that," or "Seven ways doesn't sound right; maybe six or eight or nine, but not seven." Post-solution justifications like "It's gotta be 24 ways, because I can have any of four stripes on top, and then there's six ways to put the other three stripes;" or "24 makes sense because there are 4 people who could go first, and then for each one of them there's 6 ways to put the other guys."
IID2 (Alternative solutions)	(Note that points can be awarded here only if a first solution is given.) Wants to repeat one of the colors (e.g., blue-yellow-blue), or one of the performers (e.g., have the acrobat appear twice in the same show).
IID3 (Alternative ways to solve)	Makes a comment like "There's another way you could get all six orders for the 3 stripes under the top stripe--start with PGB and keep putting the bottom stripe on top, and then do it again starting with GPB."
IID4 (Related problems)	(Note: To get points under IID4, the "related problem" must be something *new* (as opposed to some past situation merely recalled by the child) and it must be suggested by the child. (See IIB1 on recalling information.) Furthermore, it cannot be something said in response to what amounts to R's direct request to think of a related problem.) Makes a comment like "I wonder how many different shirts you could make if there were *five* different colors."

Instructions for Coding P-SCORE Behaviors for PSAs B and B'

The following are some examples of behaviors in PSAs B and B' that could be coded against certain subgoals of GOAL II:

Goals	Behaviors
IIB1 (Recall information)	Any reported recollection of information or assessment of what S has learned so far, as a result of working on the problem, so as to move forward -- e.g., "So then I asked myself if I had tried 5 cards in the older pile, and..."
	(Note that the mere recall of the problem-solving steps used, in response to R's probes, does not count here.)
	Any recollection of similar problems, whether or not they were from the pretest. (Award 2 points if S uses recalled information to solve the current problem; 1 point if the recollection pertains only to the context of the problem.) E.g., "You know, this reminds me of a puzzle that I did once where you had to put cards into piles, but those were regular playing cards" (1 point); or "I remembered that the last time you were here we did the same kind of thing, but there were 23 prices. So I tried 7 people at one table, just like last time, but it doesn't work out, so I don't think you can do it." (2 points).
IIB2 (Estimate or approximate)	Makes two stacks and compares heights; says "Well, this one looks twice as high as this one."
IIB3 (Gather info; check resources)	Asks the researcher (either immediately or after instruction or by calling researcher back) for clarification of the rules, initial instructions, or other information previously presented or implied (1 point); or asks for clarification of something that was not already stated or implied by researcher (could be 2 points). Do not award points for immediate confirmation -- "I put the cards into piles, right?" is confirmation (no points), but "Should I put the cards into piles?" is 1 point if S has already been told this, and 2 points if S has not been told.
	Explores cards by, e.g., spreading out cards to look at all the numbers; or counting all the cards to find out how many there are in all; or looking at the numbers on the cards one at a time, without putting them into piles. (Don't award points twice if more than one of these are part of the card exploration.)
	Looks at instruction chart to remind self of conditions (award only 1 point for this).

IIB4x (Calculate)	(As always, don't award points more than once for any one of the following, although multiple points can be awarded if S does more than one of them.) Does, e.g., 2 × 9 to see if 2 × 9 is equal to the number of cards in some other pile; or does, e.g., 6 + 2 = 8 and concludes incorrectly that a pile with 8 cards has twice as many as a pile with 6 cards. Adds up the ages in a pile of cards, with or without a calculator. (Give just 1 point for calculator use if S does not talk about it later.) Counts the cards in one or more piles that the S has created.
IIB4y (Manipulates)	Makes groups of cards. This can be either forming piles for the tables or making subpiles to help with ordering (e.g., putting all the 30s together).
IIB5 (Consider probabilites)	"If I just divide up the cards into three piles, with 5 cards in two of them, then I probably won't have the ages in the right order."
IIB6 (Trial & error; guess & check)	(Note: To get points under this heading, the S must realize that something is amiss, then use some method to correct the problem, and then check that the technique gives better results. A rationale must be given. Trying lots of things isn't by itself trial-and-error or guess-and-check. Nor is trying more than one approach -- e.g., first separating the cards or tags at random and then dealing them out successively into three piles.) Makes three piles (e.g., with 5, 5, and 13), checks and sees that the 5-13 split isn't correct, and then shifts some cards from one 5-pile to the 13-pile to create two new piles with a smaller difference between them, and then checks whether that does or does not satisfy the twice-as-many condition. Realizes that the older table (or one of the shelves) does not have twice as many cards as the younger table, (or another shelf) and then shifts one or more cards from the middle table or shelf (whether or not they are appropriate cards, age-wise) to one or both of the other tables or shelves; then checks to see if the older table has twice as many as the younger.
IIC1x (Picture; diagram; table; chart; list)	Makes a list of ages, separated into tables. Draws a picture of two or three tables. Writes a list of all the ages, in order.
IIC1y (Use objects; act out)	Uses three objects to stand for the three tables, and arranges the cards around or on the "tables".

	Makes signs for the three shelves indicating how many clocks are on each (e.g., subject 57).
	Personifies cards, e.g., imagines how people of various ages would look as a guide to placement of tables (see subject 11 or subject 17).
IIC2 (Transform problem)	Deliberately addresses one subpart of the problem first (e.g., "First I'll get all the cards in order; then I can decide how to split them among the tables" or "I know there are 23 cards, so I'll put them into piles of 4, 4 and 15; then I can worry about the ages.")
	Groups the cards according to decades and describes the grouping (e.g., "It will be easier to put the cards in order if put all the 40s together in one pile and all the 50s in another.") Note: Do not award points if the discussion of decades is only to demonstrate that piles meet the age/price criterion, where the piles themselves are not broken at decades.
	Simplifies the problem by considering a smaller number of cards; e.g., "Let me see if I can do the problem with only 10 cards"; or a smaller number of tables or shelves.
	Works backwards; e.g., "If I put 3 clocks on one shelf and 6 on another, then I can go from there to figure out how many clocks are left for the third shelf."
IIC3x (Look for patterns)	Seeks patterns in the numbers on the cards -- e.g., why some are repeated and some are skipped.
	Notes numerical patterns like "Every time I move one card from one pile to another pile, the difference between the numbers of cards in the piles changes by 2."
	Notes or uses a pattern related to the context that does not lead to a correct solution. E.g., groups the clock prices by odds and evens, and then forms a third shelf by taking the most expensive clocks from the two piles.
IIC3y (Pert. vs extraneous)	Notes extraneous features of the situation and rejects them; e.g., "Some of the guests are in their 60s, but the only thing that matters is that they're older than the other people. So it's OK to have a younger senior citizen sitting at a table with a bunch of 40-year-olds." Or "Some of the guests are boys and some are girls, but that doesn't have anything to do with where I put them."
IIC4 (Reapproach problem)	(Note: With one exception, to get points under this heading, a S must reject a previous approach in favor of a new one, and that new approach must be pursued to the point of deciding that the new approach is or is

not an effective one. The exception is attempting to do the optional, determining (incorrectly) that the optional part is impossible, and therefore deciding not to do it. See subject 24.

Comments like "Oh! For some reason I thought that the oldest table should have the most people at it, but now I see that isn't necessarily the case, so I'll try a smaller number of cards at that table." Or "At the beginning I thought that you should try to have the same number of clocks on each shelf, but then I realized I couldn't have it that way and still have twice as many on one shelf." Or "I thought initially that this was an old person, but then I realized she would have to go into the middle table" (see pilot subject F.Q., posttest, p.10). Or "I had one pile of 9 so I tried to make 18, but I couldn't. But then I saw this other pile had 6, so I made this one 12" (see subject 7).

IID1 (Reasonableness)	Discusses how and why the distribution of guests among (or between) the tables makes sense -- how it fulfills the criteria of the problem or fails to fulfill the optional criterion. (But don't award points here if the S is just responding to R's "How come?", "Can you show me" or "How do you know?" That is, mere recall of the steps the S went through does not constitute IID1 by itself.) (Note: Do not award 2 points if S discusses reasonableness of omitting the optional part, unless S's solution is that the optional part is impossible.)
IID2 (Alternative solutions)	(Note that points can be awarded here only if a first solution is given.) Makes, e.g., piles of 5, 10 and 11 (young, middle, old), says it's a solution; then shifts the youngest people at the middle table to the younger table (getting 10, 5, 11), and says it's another way to do it. Says, "There are lots of other ways to do this..."
IID3 (Alternative ways to solve)	Comments like: "Another way to do this would be first to put the youngest 5 in one pile and divide the others randomly into piles of 10 and 11. Then anytime you see a person who is older in one pile than someone in the other, switch them. Then they'll all get into the right pile eventually."
IID4 (Related problems)	(Note: To get points under this heading, the "related problem must be something new (as opposed to some existing activity recalled by the S) and it must be suggested by the S. Furthermore, it cannot be something said in response to R's direct request to think of a related problem.) "What would happen if you wanted three times as many people at one table as one other table?"

Instructions for Coding P-Score Behaviors for PSA C & C'

General Note on PSA C and PSA C': The "What's Wrong?" part of the problem and the "Fix It" part are parts of the same problem -- namely PSA C or C'. This means that behaviors that appear in substantially the same form in both parts should not be given points in both. For example, a Subject will frequently do the same sorts of calculations with numbers on the dice or spinners as part of finding out what's wrong with the game and again, later, as part of fixing it (or as justification for a particular approach to fixing it). In cases like this, points should be awarded for IIB4x only once, even if, in attempting to fix the game, the subject changes the numbers on the dice (or the operations on the coin) so that the calculations are being done with different numbers or operations. Even if those same calculations are being performed as part of filling in a systematic chart of all possible outcomes, they can still be thought of as "low-level individual calculations with numbers on dice and operations on coins done to obtain numerical outcomes."

Note, however, that more than 2 points could be awarded under the heading IIB4x if the Subject did other calculations for other purposes -- e.g., if she added the number of chips that each player got in a series of 3 games to determine if the game looks fair in the long run. The calculations may be the same, but the source of the numbers and operations is different, and the calculations are done for a different reason.

Notwithstanding the comments above, it is often appropriate to code behavior for IIB5 (consider probabilities) in both the "What's wrong?" and "Fix it" parts of the problem. Sometimes a child will conclude from playing the game that what's wrong with it is that it is unfair (which would get 2 points for IIB5), and then, in the second part, apply probabilistic ideas by changing the distribution of outcomes between the two players. Often this second round of probabilistic thinking seems qualitatively different from the first, and hence should be awarded additional points.

Here are some examples of behaviors in PSAs C and C' that can be coded against certain Subgoals of GOAL II:

Goals	Behaviors
IIB1 (Recall information)	Any reported recollection of information or assessment of what S has learned so far, as a result of working on the problem, so as to move forward: e.g., "So then I asked myself what number had come up the most often so far, and..."; "After I made my list of things that might be wrong with the game, I looked at it and thought about each thing, and..." (Note that the mere recall of the problem-solving steps used, in response to R's probes, does not count under this heading.) Any recollection of similar problems, whether or not they were from the pretest. E.g., "You know, this reminds me of a game that I used to play where you had to roll the dice and move men, but there the dice were

	shaped the same and they had a one through five on them;" "I remembered that the last time you were here we played a game, but it had dice. The numbers were wrong in that game, so I figured it might be the same thing this time." [See IID4 on related problems.]
IIB2 (Estimate or approximate)	Without keeping track of results, "It looks like most of the time Purple wins." Or "I must have spun these spinners thirty times by now, and it seems as if the 6 is coming up much more than the 2." Note that estimation or approximation that is not directed toward solving the problem does not count here -- e.g., "How many times did you play?" "About three" (see subject 51).
IIB3 (Gather info; check resources)	Asks the researcher (either immediately after instructions or by calling researcher back) for clarification of the rules, initial instructions, or other information previously presented (which should be given 1 point); or asks for clarification of something that was not already stated or implied by the researcher (could be 2 points). Do not award points for immediate confirmation. E.g., "They spin it 9 times, right?" is confirmation (no points) but "How many times do they spin it?" is 1 point if S has already been told, and 2 points if S has not yet been told.) Rolls dice and flips coin in an attempt to generate data (whether or not S keeps track of what he or she is getting). Examines the pieces of the game by doing one or more of the following (except in response to R's direct prompt "Why don't you look at these?"): Handles the die or dice to see what the numbers are, or to count the number of sides; looks at the spinners; counts all the chips; looks at or measures the players or their signs; examines the 1-30 chart (if not as part of finding a particular number while playing); etc. (Note: Simply referring to number chart (or to the signs on the players) does not constitute an example of IIB3, since that's part of playing the game.)
IIB4x (Calculate)	Performs additions or multiplications of numbers that appear on dice or spinners, with or without a calculator. (Note: Do not award points for calculations done in response to Researcher's initial illustrative examples, unless they are spontaneously generated by the S.) (Note: Award 2 points if a full game is played, under the assumption that calculations are being done, unless there is reason to think otherwise.)

Counts, or performs additions or multiplications in connection with analysis of probabilities -- e.g., "There are 3 ways for Orange to win and 9 for Green."

IIB4y (Manipulate)

Plays the game -- i.e., rolls dice or spins spinners, flips coin, moves chips in front of players. This is sometimes done mentally, when fully described as genuine game play, not just generating data. (Points should be awarded at most once even if a second game is played following some fixing; points under IIB6 might be given for the second playing.)

Physically changes (or describes physical changes in) the game by doing one or more of the following (award points under this heading for making changes like these at most once):

- o changes the numbers on the dice by putting stickers on them or by pretending that one number is some other number;
- o changes the numbers on the spinners or the amount of spinner devoted to a particular number by using stickers or by drawing or writing directly on the spinners;
- o changes the operation(s) on the coin;
- o moves the colored loops so that they enclose different sets of numbers;
- o adds more numbers to the chart physically (e.g., via stickers) or mentally (e.g., by saying "I'd put a 36 here in the green loop.")
- o measures the chips to note imperfections.

IIB5 (Consider probabilities)

Any statement that includes "probably" or "better chance" or "unfair"

Statements like "The purple guy wins more than the red guy" if it is a probabilistic statement rather than one of empirical fact; or like "I kept doin' this [rolling dice] to see what would mostly come out" (pilot subject C.M., pretest, p.6)

(Note: A statement of certainty, like "The times guy is always going to land on more-than-ten" (pilot subject J.R., pretest, p. 3) is not IIB5, despite the interpretation of probability = 1. See IIC3.)

(Note: Remember that points can be awarded for IIB5 in both the "What's Wrong?" and the "Fix It" parts. Also, points within the "Fix It" part can be awarded more than once if a different instance of probability is involved. E.g., pilot subject C.M. (pretest, pp. 11-12) notes that the game is unfair and then uses probability to determine which numbers to change.

IIB6 (Trial & Error; Guess & Check)	(Note: To get points under this heading, the S must realize that something is amiss, then use some method to try to correct the problem, and then check that the technique gives better results. Trying lots of things isn't by itself trial-and-error or guess-and-check. Nor is trying more than one approach -- e.g., changing the numbers on the dice and then moving the colored loops. See IIC3 and IIC4.)
	Changes the cut-off point to 14, e.g., and then checks this decision either by playing the game again or by some more systematic analysis of how the obtainable sums and products compare with 14.
	Moves the colored loops to correct a perceived unfairness, e.g., and then plays the game to see if the results are fairer.
	Changes some numbers on the dice, and then moves the dice around to check that more combinations of numbers are now available, thus making it a more interesting game.
IIC1x (Picture; diagram; table; chart; list)	Records or lists the results of game play -- either just sums and products, or numbers leading to those sums and products, whether or not associated with a player. Even a simple tally of number of times played would count here.
	Makes a table or chart whose purpose is to show all possible outcomes of dice/spinners (whether or not it in fact does so).
	Writes a list of things that are wrong with the game.
	Draws a picture of the dice, or other parts of the game.
IIC1y (Uses objects; acts out)	Personifies the players; e.g., "The orange man is happier now because he has the same chance of winning."
	Moves the players next to their colored rubber bands and says "This way they will be closer to their numbers and they'll be able to keep track of their numbers better."
	Turns the pointers (rather than spins them) to point to particular numbers to stand for a potential spin's outcome; or, similarly, turns the dice. (Note that this would not be counted as generating data or checking resources, IIB3.)
	(Note: The use of fingers or a calculator for calculating does not count as IIC1y.)
IIC2 (Transform problem)	Considers subproblems by temporarily fixing the value of one of the random devices; e.g., "Suppose this die is always 5; then what could the other things be?" or (as part of explaining why the game isn't fair) "If you just had plus it would be fair."

Works backward; e.g., "For it to be fair there would have to be six numbers in each rubber band, so then I had to figure out which numbers I could put on the spinners to get it that way."

Works backwards from the numbers on the board to the spinners -- e.g., "There was no numbers [on spinners] that go into [result in a sum or product of] 17 or some of the other numbers" (subject 51).

Clarifies the problem; e.g., "I moved the man with the orange sign right next to the orange side of the number thing, and the green man too. That made it easier."

IIC3x (Look for patterns)

Looks for or identifies patterns like "Every time the 8 comes up, the purple guy wins." "Because if you get times, you would get more than 10." "All the multiples of 6 are in the column on the right." (Note that the alleged pattern need not be completely correct. Note also that the pattern need not be described verbally -- e.g., if S plays the game but doesn't flip the coin unless the two numbers obtained are such that the coin outcome would affect which player is awarded a chip.)

IIC3y (Missing info; pert. vs extraneous)

(Note: To get points for "pert vs extran", the S must reject some well-formed idea as being based on some extraneous aspect of the problem situation, although evidence is not needed to support this position. S may or may not consider something else as relevant, instead, and that may or may not be in the same domain. Examples: "The shape of the men" or "The color of the men" could be rejected as extraneous, but not just "the men"; cf. pilot subject M.O., posttest, p. 24.)

(Note also that IIC3y can often arise when S is asked if there was some idea that he or she rejected; we can often infer that the idea was rejected simply because it was given in response to that question.)

Distinguishes between pertinent and extraneous aspects of a situation (e.g., "The color of the men can't have anything to do with it"; "I'm sure the shape of the dice isn't important, but I think the numbers are." Or expresses a well-formed hypothesis, tests it, and rejects it (e.g., subject 57 thinks one player might have more numbers on the board, counts numbers to find 15 each, and rejects that as a source of unfairness). (As usual, award points under this paragraph at most once.)

Writes, but does not ask, a question like "Can they switch tags?" (see pilot subject H.L., pretest, p. 11).

IIC4 (Reapproach problem)

(Note: To get points under this heading, the S must reject a previous approach in favor of a new one, and that new approach must be pursued to the point of deciding that the new approach is or is not an effective one. If there's any doubt as to whether a behavior should be coded as IIC4 or IIC3y, code as IIC3y.)

	(Note also that, e.g., changing multiplication to subtraction and then again to division is not reapproaching the problem, although it might be IIB6 if sufficient checking occurs.) Comments indicating a new approach, like "At first I thought it must be the dice because they're shaped funny. But then I realized it wasn't the shape as much as the fact that there were too many 8s. So then I..."
IID1 (Reasonableness)	(Note: Don't award points here if the S is just responding to the Researcher's "Tell me what you did" or "How do you know that?" I.e., mere recall of problem-solving steps is not IID1 per se.) Justifies conclusion that game is wrong (e.g., unfair) through reference to data gathered or analysis completed. Justifies method of fixing game through reference to steps taken to fix it (e.g., "Now it's a more interesting game because I put six other numbers on the funny die and that will give you more combinations to do when you play it.")
IID2 (Alternative solutions)	(Note that points can be awarded here only if a first solution is given.) First changes numbers on the dice, then says "You know there's another thing that makes the game wrong -- the cut-off score is too low." [This is an alternative solution to the problem "What's wrong with the game?" See IID3, below.] (Of course the S could get points under other headings in connection with each of the solutions.) Or, first says that one of the chips is defective, and then says that another thing that is wrong is that there are too many nails in the board.
IID3 (Alternative ways to solve)	First changes numbers on the dice, then says "You know you could fix the game another way -- by changing the cut-off score." [This is an alternative way to solve the problem "Dr. Game wants you to fix the game." See IID2, above.] (Note: If there is any doubt as to whether IID2 or IID3 is the appropriate category, code as IID2.)
IID4 (Related problems)	(Note: To get points under IID4, the "related problem" must be something new (as opposed to some existing game merely recalled by the child) and it must be suggested by the child. Furthermore, it cannot be something said in response to R's direct request to think of a related problem (see, e.g., pilot subject S.M., pretest, p. 17, which is not IID4). (Note: If a child finds something wrong with the game, and then finds something else wrong with the game, this does not constitute an example of IID4, although it may be IID2.)

Creates a related game, or fixes the game as originally presented, as part of explaining what's wrong with the original game -- e.g., "It would be better with subtraction and division."

Creates a related game as an extension of the original one -- e.g., "I wonder if I could make up a fair game that has 3 players."

APPENDIX II.H:

M-score Coding Guide

INSTRUCTIONS FOR CODING M-SCORE FOR PSA A and PSA A'

There are three basic components of problem-solving activities A and A'. The first is to find as many ways as possible to order the shirt stripes or circus performers; the second is to explain how one knows that all the orders have been found; and the third is to explain why no orders have been repeated. Each of these components is scored independently according to the guidance below. Then the S's score for "how far he or she got into the problem" is just the sum of scores on the three components.

This guide should be read in conjunction with the flowchart on page 4. Each row represents one of the three basic components of the PSA.

Find as many different orders as possible

A. +0 No understanding of the problem; no new orders beyond the ones created with the researcher.

B. +1 Some minimal understanding of the problem with at least one order (other than the starting one) produced. No system for generating new ones, no system (beyond unsystematic memory) for recording ones already produced.

OR

Some understanding of the problem, with some different orders created, but tied inappropriately to the context of the problem -- e.g., "Blue is the only color that could go on the top because it looks better that way."

C. +2 Some systematicity as evidenced by some repetition of an action. E.g., S puts top stripe on the bottom and then puts the new top stripe on the bottom. Or S switches the top two stripes, and then, starting from a third order, again switches the top two stripes.

OR

Blind unguided generation of orders followed by checking (memory or paper) to see if it has already been done.

OR

Six or more different new orders, generated with no apparent system.

D. +3 A more extended and explicit system for generating different orders, which, if followed consistently, would produce at least four different orders. E.g., "I just keep putting the first performer last" (which would generate exactly four orders) or "I switch the first two, then the last two, and then the middle two" (which would generate 12 different orders); or "There are four cards and four positions for each card, so it's 4 × 4, or 16."

OR

Any systematicity that results in at least 12 different orders, even if the system is not followed perfectly consistently.

OR

Some systematicity as shown by the explicit use of previously generated orders to create new ones. E.g., "I looked at the ones I had already and made the next new one be different in at least one position from anything that I had before."

E. +4 An even more extended, explicit, system for generating orders, where the aim is to get all the orders. Example: "I put the red on top and got all the ways I could do it that way, and then I put blue on top..." Or: "I got 6 with brown on top, and 6 with pink on top [not all four top stripe colors]."

F. +5 All 24 orders, whether or not they are all explicitly listed.

How do you know you have them all?

G. +0 Doesn't know

H. +1 Empirical justification -- e.g., "I tried lots of ways and this is all I could find". Good evidence for a score no higher than this is given by a subject who thinks that someone else might be able to find more orders (see, e.g., pilot subject S.M., pretest).

OR

Was interrupted and realizes that there are more to be found.

I. +2 Theoretical justification, even if the details aren't quite right. E.g., "If two orders are different then they have to be different in every place. Since there are only four people, there can be only four different orders." Or "With the blue stripe on top there are five different orders. So there would the five with yellow on top, and so on. So there are a total of 4 × 5, or 20, different orders." Or "4 performers -- 4 × 4 is 16 -- so there's 16 ways."

J. +3 A full, correct justification -- e.g., "There are 4 ways to fill the first spot, 3 to fill the second, 2 for the third and 1 for the last. So there are 4 x 3 x 2 x 1 or 24 different orders." (The "x 1" part of it is not required.) Or "If there were only three stripes, there would be 6 different orders. So for each of the four possible top stripes there are 6 ways to put in the 3 lower stripes, so that's 6 x 4, or 24, ways in all." Or "Once you have 3 performers in some order, then there are 4 ways to insert the fourth person. So since there are 6 ways to do 3 performers, there are 4 x 6 ways to put 4 performers."

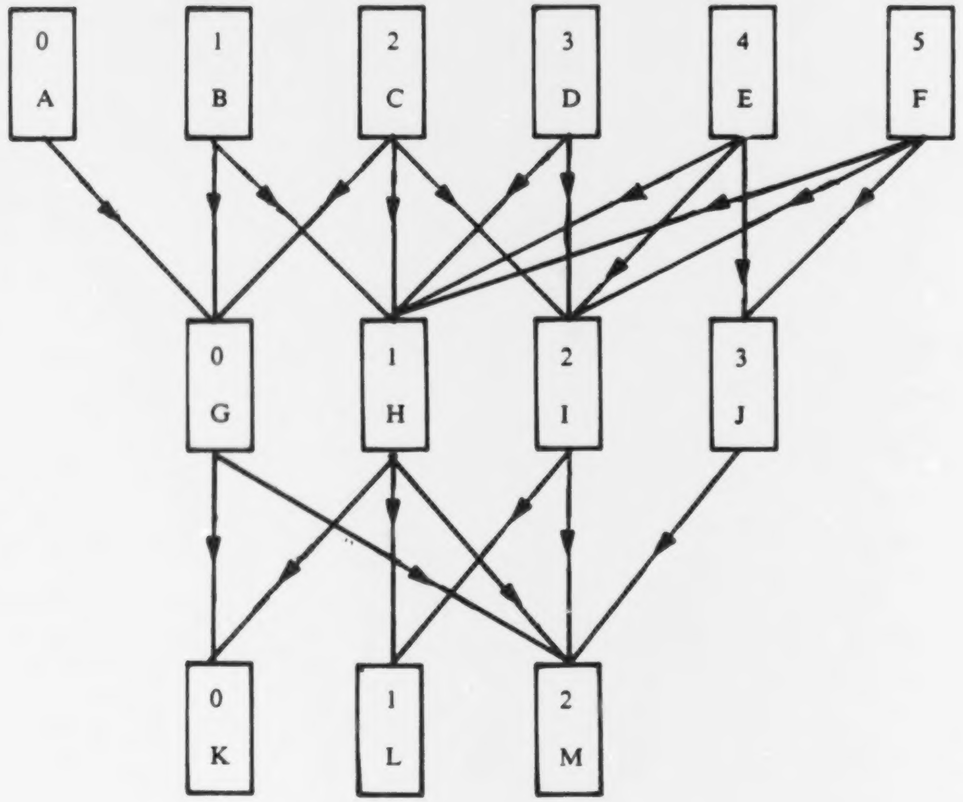
How do you know you didn't repeat any?

K. +0 "I don't."

L. +1 Empirical justification -- e.g., "I looked at all the orders I made and didn't see any that were the same."

M. +2 Theoretical justification -- e.g., "It's impossible for there to be a repetition because the method I used guarantees that all the orders are distinct." Note that the "method" used could be the consistent checking of new orders against the list of orders already generated.

FLOWCHART for the components of PSA A and PSA A'



As
many
orders as
possible

How do you
know that
you have
them all?

Repeti-
tions?

INSTRUCTIONS FOR CODING M-SCORE FOR PSA B and PSA B'

There are four basic components of PSAs B and B': One is to make three piles of cards (guests or clocks); a second component is to have at least 5 (or 4) in each pile; a third is to satisfy the order condition (having a more expensive shelf or older table, etc.); the fourth component is, optionally, to make one pile have twice as many cards as one of the others. These four components are independent in the sense that any one can be achieved without the others, and so they should be scored independently. The S's score for "how far he or she got into the problem" is just the sum of his or her scores on the four components.

See the flowchart on page 8.

Correct number of groups

A. +0 Some number other than 3.

B. +1 Exactly 3 groups of cards.

Minimal number of cards in each group

C. +0 At least one pile has fewer than the minimum number.

D. +1 All piles that are supposed to represent shelves or tables have the minimal number required. (Note that if S has neglected to include some cards among any of the shelves or tables, those cards should not be counted as a shelf or table that doesn't meet the minimal cards condition.)

Ordering the cards

E. +0 Is unable to do anything that takes into account the order relations among the cards; ignores this component entirely.

F. +1 Compares the cards two-at-a-time, and notes that one or more cards violate the ordering condition of the problem, but is unable to segregate the cards into three piles where most of the cards in the piles meet the age (or price) criterion.

G. +2 Is able to separate the cards so that with at most two exceptions all the cards meet the age or price criterion. (The problem may be in selecting the particular card(s) to move from one pile to another to adjust the number of cards in the piles.)

H. +3 All the cards that are in the piles for shelves or tables (regardless of how many groups there are) have the proper order relations, regardless of the number of cards in each pile.

Justifications for ordering. Note that for many subjects it is completely obvious that the groups they have created satisfy the age/price ordering criterion; their use of transitivity is so immediate and natural that they have difficulty in understanding the meaning of the question. Some subjects will respond simply by saying "Because the numbers are bigger."

I. +0 None

J. +1 Vague indication that some of the cards in one pile are older than some of the cards in some other pile -- e.g., "This is the table for the older people" or "The cheaper clocks are in this pile."

K. +2 A justification that depends on an initial ordering of the complete set of cards -- e.g., the S puts all of the cards in order and then splits them into three sets.

OR

A justification that involves a correct further breakdown of the ages or prices -- e.g., "I put all the twenties and thirties on the low price shelf, the forties and some of the fifties on the medium shelf, and the rest on the expensive shelf.

OR

A justification that implicitly involves transitivity of order -- e.g., "The oldest person at this table is 37, and she's younger than anyone at these [other] tables."

OR

A justification of a correct ordering that seems to miss the subtlety of the question -- e.g., "Just look at the numbers," or "This was the most expensive shelf because all these numbers are bigger than all the other numbers."

OR

A justification that is in effect an exhaustive comparison of each card in a particular pile with all the other cards -- e.g., by showing the researcher each card (e.g., see pilot subject M.O., pretest).

Meeting and justifying the "twice-as-many" condition

L. +0 Does not attempt to meet the optional condition, or was interrupted before having the opportunity to try it, or doesn't want to try it.

P. +1 Considers the optional part, but does not pursue it for some stated reason other than mere difficulty or optionality.

M. +1 Does attempt the optional part and makes or considers making one pile larger than the other, via

+0 global impression
+1 careful eyeballing of stack heights
+2 counting of cards in each pile (even if inaccurately)

and says that the larger pile has "twice as many" cards as the smaller pile.

OR

Makes one pile with 10, claiming that that's twice as many as the minimum number (5), even though no other table has 5 (see subject 55 or 29).

Makes one pile with five cards (e.g.,) and later adds another five to it, claiming that now this shelf has twice as many clocks as it had before.

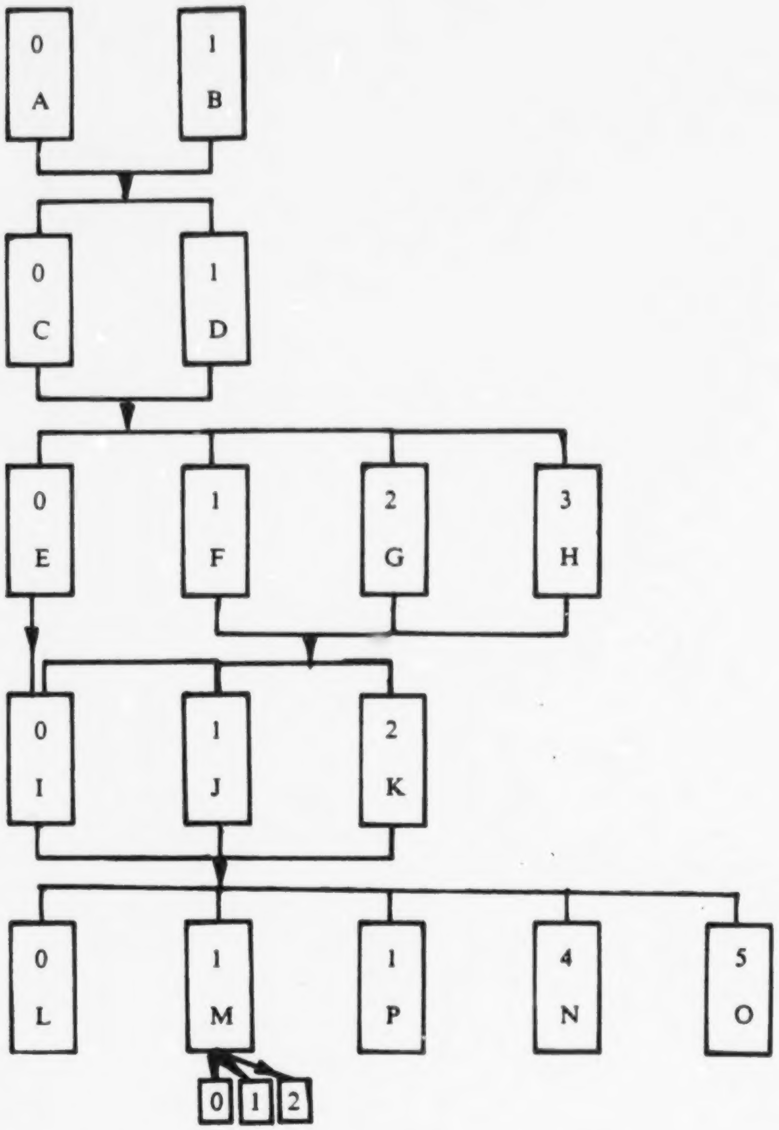
OR

Makes, or tries to make, the sum of the ages (or prices) in one pile twice the sum of ages or prices in another pile.

N. +4 Makes, or considers, two piles, in an incorrect split, and spontaneously realizes that the twice-as-many condition has not been met (e.g., because "12 is not twice as many as 7"); but is unable to adjust the piles appropriately. (Note that this level requires counting of the two piles and must not be merely the result of S's perception of R's doubt.)

O. 12 +5 Makes, or considers, two piles in a correct split, e.g., a 6-12 split, and says that is twice as much as 6. (Note: Do not award any points if the S has a correct distribution of cards but doesn't realize it or claims that he or she did not attempt the optional part.)

FLOWCHART for PSA B and PSA B'



Number of groups

Minimum in each

Ordering cards

Justify ordering

Optional condition

Instructions for Coding M-SCORE for PSA C, with Dice

PSAs C and C' are sufficiently dissimilar that we treat them separately. Both PSAs consist of two basic parts: determining what is wrong with the game, and then fixing it. Each part is scored independently, according to the guidance set forth below. A S's M-score is then the sum of his or her scores on the subparts.

Note that this document should be read in conjunction with the schematic flowcharts on pages 12 and 15. Each row of boxes in the flowchart represents an opportunity to attain points. In each case the S's most sophisticated behavior in each row is the one that should be scored. In particular, if a S suggests more than one way to fix the game, he or she should get points only for the most advanced or well developed fixing technique offered.

What's wrong with the game (and why?)?

A. +0 Don't know; or nothing's wrong.

B. +1 Any physical feature of equipment (e.g., color, shape of die or dice, numbers on dice; or color, shape, number of players; state of disrepair of any of the equipment) if the reason why is that it's different from, or doesn't conform to, the S's idea of what dice, coins, etc. should be like or were in fact like originally. (See also C and D, however.)

C. +2 Ditto, including the distribution of numbers on the dice, if reason is that it doesn't meet child's criterion for a worthwhile "math game" -- e.g., it should have minus or divided by (to get practice in these operations) or more numbers so that a larger variety of arithmetic exercises would appear (to avoid boredom or to gain practice).

D. +3 Purple wins a lot (not wins more than Red).

OR

The distribution of the numbers, with a vague or poorly specified connection between that and the outcomes for the players. E.g., "There are too many 8s on the octahedral die and Purple has the larger numbers."

OR

The game is unfair, but not specified which player is favored.

E. +4 Any statement to the effect that Purple is generally favored, going beyond the particular game or games that were played (if any). E.g.:

"The game isn't fair to Red"; or
"Purple wins more than Red"; or
"Purple wins most of the time [i.e. more than half the time]"; or
"Purple has a better chance."
"Most of the time it will probably come out to be more than 10" [and variants].

WHY? (following D or E, above)

F. +0 Don't know; or no reasons; or "It's just luck" or "Purple is luckier"; or just repeats statement that the game is not fair.

G. +1 Any reason limited to the game(s) just played, e.g., "That's what happened when I played it, and the score was 7 to 2." This includes any number of individual isolated unconnected examples, whether from the game just played or made up on the spot.

OR

Any reason beyond luck.

H. +2 A more general statement about how the game favors Purple, not limited to the game(s) just played; e.g.,

"There are more ways (or chances) for Purple to win [a chip]"; or
"Purple gets more of the numbers"; or
"Most of the numbers will be big, and that means that Purple will win those."
"Most of the time the numbers will be more than ten," if the connection between that and the unfairness to Red is also made.

(Note that some of these might also get points later on.)

WHY? (following H, or even G, above)

J. -1 There are more (whole) numbers larger than or equal to 10 than there are (positive whole) numbers less than 10. (This can follow H only, to give a total of 1.)

OR

There are more whole numbers from 10 to 32 [the largest possible sum or product] than there are from 1 to 9 (and, in particular, obtainable numbers from 1 to 9).

K. +0 Don't know; or "It's just luck"; or no legitimate reason.

L. +1 Some examples of ways in which Purple (and, possibly, Red) could win, chosen randomly with no interconnections. I.e. more winning combinations for Purple come to mind readily than winning combinations for Red. (Don't award unless examples are spontaneously generated by S.)

OR

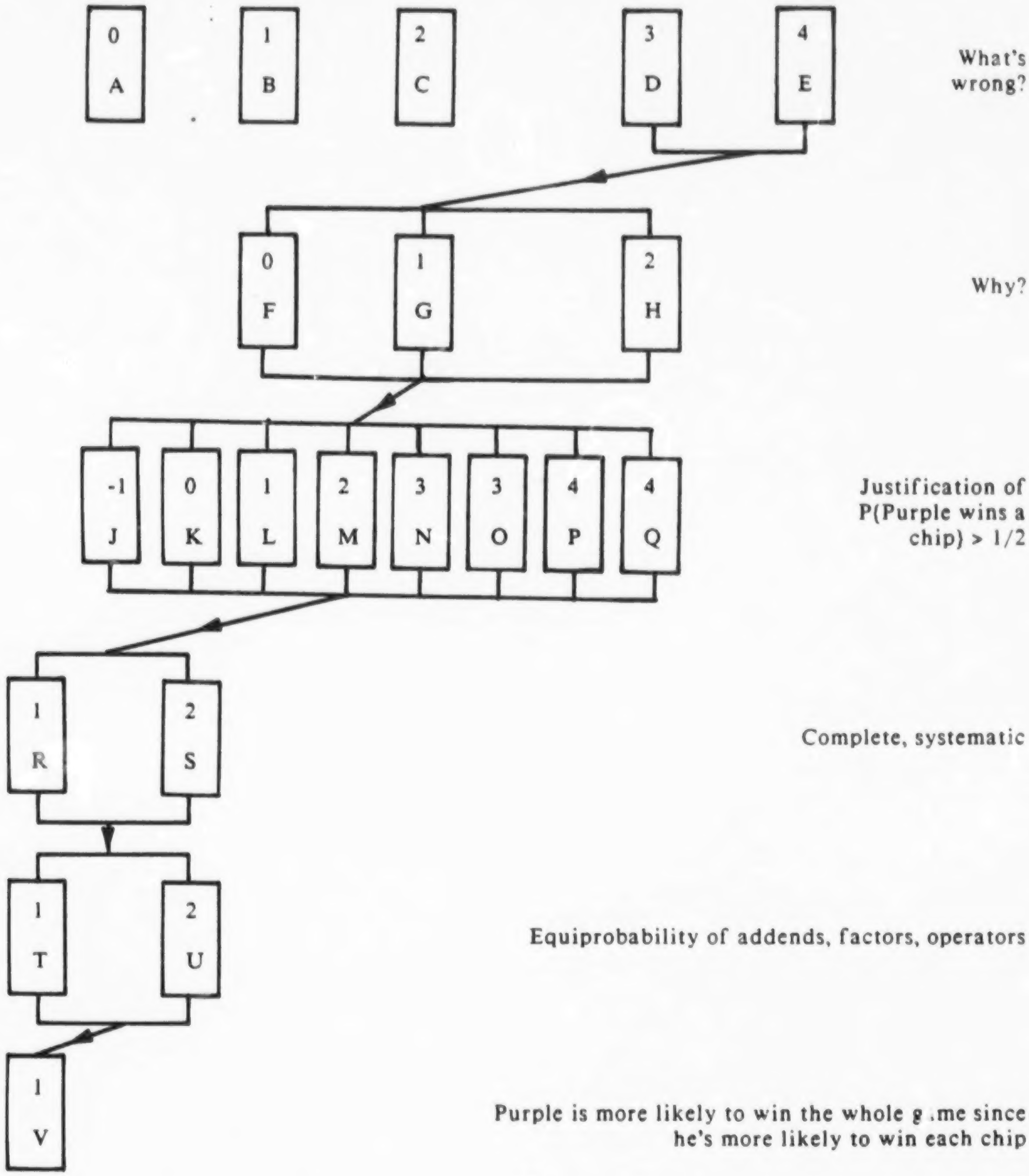
Any assertion that the numbers on the dice are large, and so sums and products will be 10 or more, without specific examples.

M. +2 Some winning examples for Purple, with some partial interconnections -- e.g., "Purple could win with 8, +, 2-or-3-or-4; or 8, ×, 2."

N.	+3	Complete, but nonsystematic, coverage of all cases for Purple and Red, with at most one omission or duplication.
O.	+3	One correct statement of the form "Purple wins every time [something or other] happens," where the adverbial clause comprises two or more combinations. E.g., "Whenever the times comes up, Purple wins," or even "Purple wins whenever the 5 comes up, unless the coin is plus."
P.	+4	Two or more distinct statements of the kind listed in O., above.
Q.	+4	Any set of statements that prove (assuming equally likely outcomes) that P(Purple wins a chip) > 1/2, even if they don't show that P(Purple wins a chip) = 3/4. E.g., "Purple always wins if the 8 comes up (and that's half the time) and Purple sometimes wins even if the 5 comes up" or "If it's plus, then Purple and Red have the same chance, but if it's times, then Purple wins." This can be awarded even if the analysis is based on obtainable sums and products (not taking into account the probability of each result -- e.g., that getting a 10 is twice as likely as getting an 11).
R.	+1	Complete systematic coverage of all 12 possible outcomes, even if they're not counted or considered individually. E.g., "Red wins if it's 5 and +, and Purple wins in every other case." OR Complete, systematic coverage of all possible obtainable sums and products, even if they are not assigned to correct probabilities.
S.	+2	Complete coverage of all 12 possible cases, with counting. E.g., "There are 3 ways for Red to win and 9 ways for Purple." Also consider this attained if the cube's 2-3-4 are treated as a lump -- e.g., "There's one way for Red to win (getting a 5 and a plus) and 3 ways for Purple (i.e. 5 & ×, 8 & +, 8 & ×).
T.	+1	The equivalent of one of these: "5 and 8 are equally likely because the octahedron has four 5s and four 8s, and each face is equally likely to turn up" OR "2, 3, and 4 are equally likely because the cube has two of each number and each face is equally likely" OR "Plus and times are equally likely."
U.	+2	All of the points listed above in T.
V.	+1	"Since Purple is more likely to win each chip, he's more likely to get more than 5 of the 9 chips than Red is."

PSA C FLOWCHART for "What's wrong and why?"

Directions: The child's score is the sum of the highest number in each row. See previous description for what the letters are for.


0
A
1
B
2
C
3
D
4
E
What's wrong?
0
F
1
G
2
H
Why?
-1
J
0
K
1
L
2
M
3
N
3
O
4
P
4
Q
Justification of P(Purple wins a chip) > 1/2
1
R
2
S
Complete, systematic
1
T
2
U
Equiprobability of addends, factors, operators
1
V
Purple is more likely to win the whole game since he's more likely to win each chip

How would you fix the game?

	FROM	Score	
A'	A, B, C	0	Any change of the irrelevant physical features of the game. OR Any change of relevant features (e.g., the numbers on the dice) for an irrelevant reason (e.g., it makes a more interesting game).
B'	G, H	+1	Any change (e.g., making the numbers on the dice lower) that is designed to make the game fairer (as opposed to make it more like more familiar dice games).
C'		+1	Change of operation, replacing × with - or / or +, again designed to make the game fairer, (as opposed to making it more challenging computationally).
D'		+1	Raising the cut-off score.
E'		+1	Change of number of chips awarded (e.g., 2 chips to Red if the sum or product is < 10).
F'	B'	+2	Partial listing of how changes of the numbers on the dice would create additional ways for Red to win. E.g., "If I change the 4s to 1s, then 5 & ×, and 8 & + will win for Red." Or a more general characterization like "If I make the numbers on the cube 1 through 4, then the numbers will be smaller and Red will win more often."
G'		+5	Complete enumeration of how proposed changes would affect the set of possible sums and products, even if equally likely occurrence of numbers on faces of dice is destroyed (e.g., changing just two of the four 8s on the octahedron to 1s.)
H'	C'	+2	Partial listing of changes to operations, e.g., "8 - 3 will now win for Red."
I'		+5	Complete enumeration -- e.g., + is as before, but now Red wins whenever the minus appears.

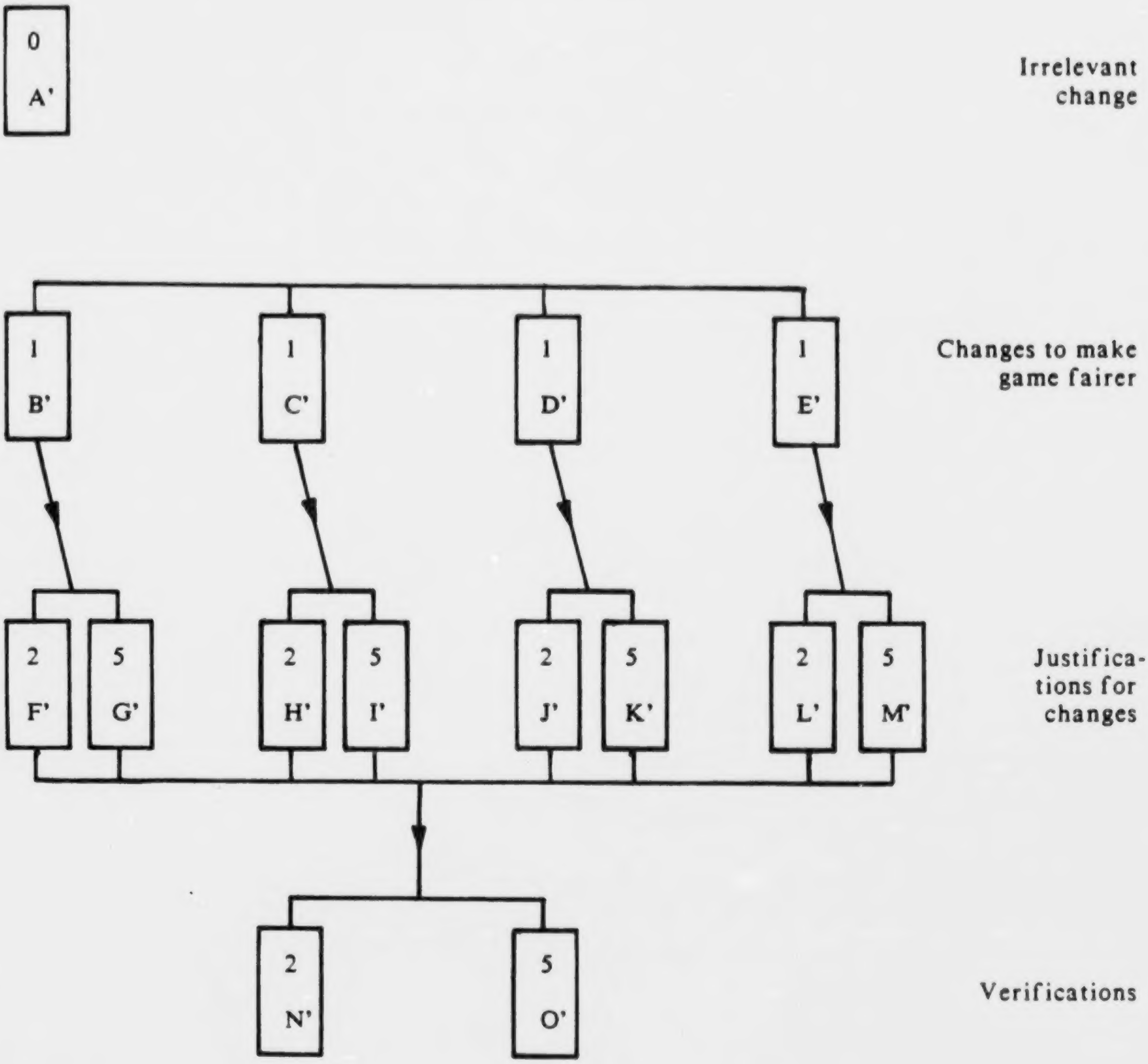
J'	D'	+2	Partial account -- e.g., "If the cut-off score is 11 or above, then 5, ×, 2 will now win for Red."
K'		+5	Complete account -- e.g., "If the cut-off score is 11 or above, then these are the winning rolls for Red and...."
L'	E'	+2	Partial account -- e.g., "If it comes up 5, +, 3, then Red will now get two chips instead of just one.
M'		+5	Complete account -- e.g., "Red will get two chips whenever the 5 and the + come up and...."

(Note that for some subjects the verification stage below will be included in the justification stage above.)

N'		+2	Verification that whatever suggestion has been made actually makes the game fairer, i.e., a verification that (at least in some cases) the game is now fairer than it was. (This includes an empirical justification -- playing the game again, with the changes, to see if it turns out to be fairer.) (Note: It may happen that the game is now biased toward the other player, but 2 points can still be awarded here.)
O'		+5	Correct verification that the suggestion actually makes the game completely fair, i.e., a verification that Red and Purple are equally likely to win.

NOTE: Any repeated attempts at verification, followed by readjustment of numbers or whatever, would be recorded as part of the child's P-score, not here.

Flowchart for How would you fix the game?



Instructions for coding M-SCORE for PSA C', with SPINNERS

NOTE: Please see the introductory notes for the PSA C M-Score (Mathematical sophistication and correctness) coding description.

This document should be read in conjunction with the schematic flowcharts on pages 20 and 23.

What's wrong with the game (and why?)?

A. +0 Don't know; or nothing's wrong.

B. +1 Any physical feature of equipment (e.g., color, shape of spinners or other pieces; numbers or placement of numbers on spinners; or color, shape, number of players; state of disrepair of any of the equipment) if the reason why is that it's different from, or doesn't conform to, the S's idea of what spinners, coins, etc. should be like or were in fact like originally. (See also C and D, however.)

C. +2 Ditto, including the distribution of numbers on the spinners, if reason is that it doesn't meet child's criterion for a worthwhile "math game" -- e.g., it should have minus or divided by (to get practice in these operations) or more numbers so that a larger variety of arithmetic exercises would appear (to avoid boredom or to gain practice).

D. +3 Orange wins a lot (not wins more than Green).

OR

The distribution of the numbers, with a vague or poorly specified connection between that and the outcomes for the players. E.g., "There are too many 2s on one spinner, and Orange has the smaller numbers."

OR

Game is unfair, but not specified who is favored.

E. +4 Any statement to the effect that Orange is generally favored, going beyond the particular game or games that were played (if any). E.g.:

"The game isn't fair"; or
"The game isn't fair to Green"; or
"Orange wins more than Green"; or
"Orange wins most of the time [i.e., more than half the time]"; or
"Orange has a better chance"; or
"Most of the time it will probably come out to be less than 16" [and variants].

OR

Any statement to the effect that Green is generally favored (although this is of course not true).

OR

Any statement that Green and Orange are equally favored (which is not true either).

WHY? (following D or E, above)

F. +0 Don't know; or no reasons; or "It's just luck" or "Orange is luckier"; or just repeats statement that the game is not fair.

OR

...in the case that S claims that Green is favored, that (e.g.) there are several numbers in the orange loop that cannot be obtained as the product or sum of any pair of spinner numbers.

G. +1 Any reason limited to the game(s) just played, e.g., "That's what happened when I played it, and the score was 7 to 2." This includes any number of individual isolated unconnected examples, whether from the game just played or made up on the spot.

OR

Any reason beyond luck.

H. +2 A more general statement about how the game favors Orange, not limited to the game(s) just played; e.g.:

> "There are more ways (or chances) for Orange to win [a chip]"; or
> "Orange gets more of the numbers"; or
> "Most of the numbers will be small, and that means that Orange will win those."
> "Most of the time the numbers will be less than 16" if the connection between that and the unfairness to Green is also made.

(Note that some of these might also get points later on.)

WHY? (following H, or even G, above)

J. -1 There are more numbers on the board in the orange loop than there are in the green loop (which is not true).

OR

There are some numbers in the Orange loop that cannot be obtained as a sum or product of any pair of spinner numbers.

OR

There are the same number of numbers in the green and orange loops on the number board.

K. +0 Don't know; or "It's just luck"; or no reason.

L. +1 Some examples of ways in which Orange (and, possibly, Green) could win, chosen randomly with no inter-connections. I.e., more winning combinations for Orange come to mind readily than winning combinations for Green. (Don't award unless examples are spontaneous.)

OR

Some example of a number in the Green loop that is unobtainable as a sum or product of spinner numbers.

OR

Any assertion that the numbers obtained are generally small.

M. +2 Some winning examples for Orange, with some partial interconnections -- e.g., "Orange could win with 6, +, 3-or-4-or-5; or 2, ×, 6."

N. +3 Complete, but nonsystematic, coverage of all cases for Orange and Green, with at most one omission or duplication.

O. +3 One correct statement of the form "Orange wins every time [something or other] happens," where the adverbial clause comprises two or more combinations. E.g., "Whenever the plus comes up, Orange wins," or even "Orange wins whenever the 6 comes up, unless the coin is times."

P. +4 Two or more distinct statements of the kind listed in O., above.

Q. +4 Any set of statements that prove (assuming equally likely outcomes) that P(Orange wins a chip) > 1/2, even if they don't show that P(Orange wins a chip) = 3/4. E.g., "Orange always wins if the 2 comes up (and that's half the time) and Orange sometimes wins even if the 6 comes up" or "If it's times, then Orange and Green have the same chance, but if it's plus, then Orange wins." Award this even if the analysis is based on obtainable board numbers; e.g., "The only numbers you can get are 6, 7, 9, 10, 11, 18, 24 and 30, and only three of those are in the Green loop (see Subj. 3.).

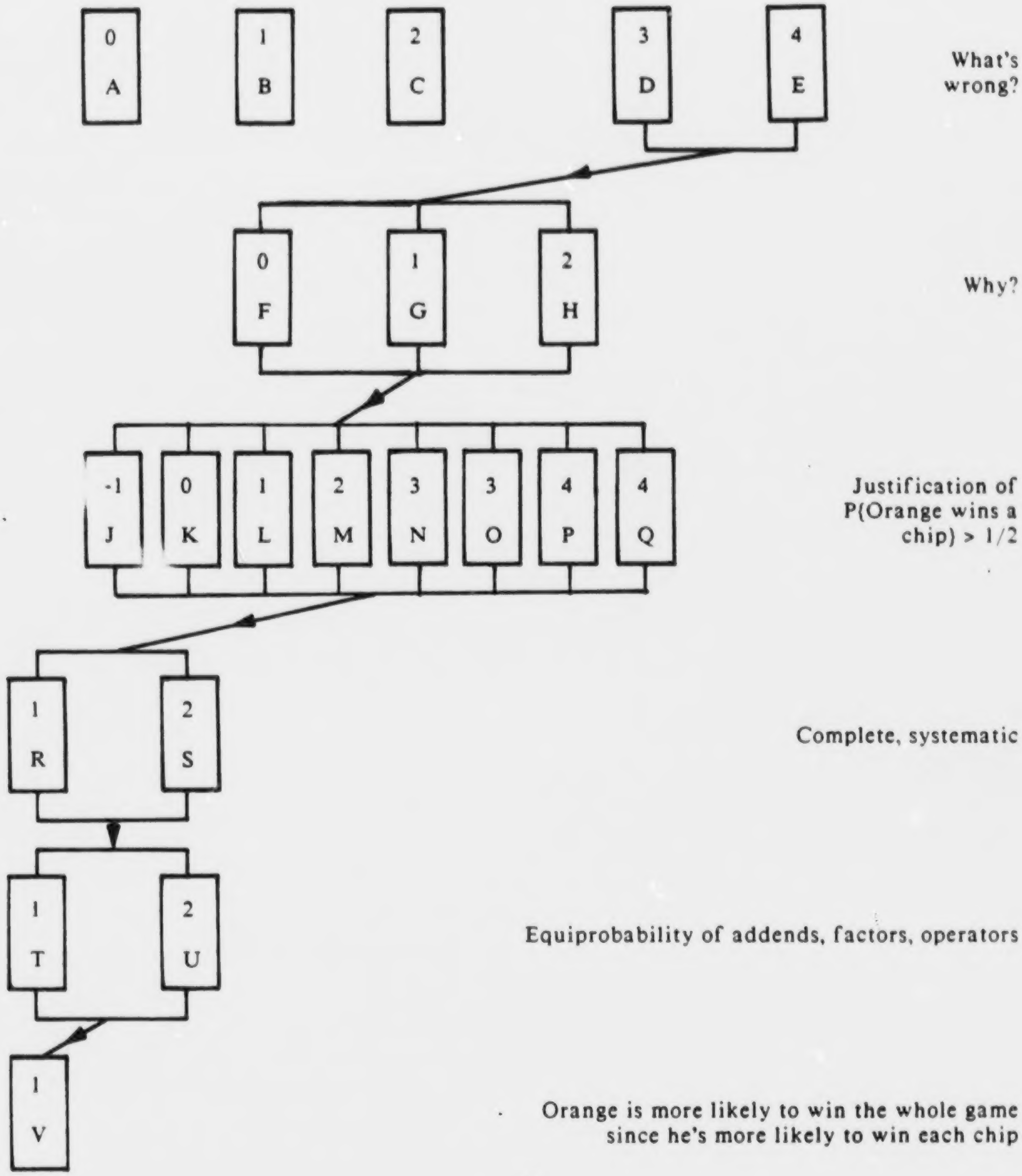
R. +1 Complete systematic coverage of all 12 possible outcomes, even if they're not counted or considered individually. E.g., "Green wins if it's 6 and times, and Orange wins in every other case."

OR

		Complete, systematic coverage of all possible obtainable board numbers, even if they are not assigned the correct probabilities (e.g., 10 is treated as equally likely as 11) (see Subj. 3).
S.	+2	Complete coverage of all 12 possible cases, with counting. E.g., "There are 3 ways for Green to win and 9 ways for Orange." Also consider this attained if the cube's 2-3-4 are treated as a lump -- e.g., "There's one way for Green to win (getting a 6 and a times) and 3 ways for Orange (i.e., 6 & +, 2 & +, 2 & ×).
T.	+1	The equivalent of one of these: "2 and 6 are equally likely because one spinner has three 2s and three 6s and each section is the same size (and therefore equally likely to pointed to)" OR "3, 4, and 5 are equally likely because the spinner has two of each number, and each [numbered section] is the same size (and therefore equally likely to be pointed to)" OR "Plus and times are equally likely."
U.	+2	All of the points listed above in T.
V.	+1	"Since Orange is more likely to win each chip, he's more likely to get more than 5 of the 9 chips than Green is."

PSA C' FLOWCHART for "What's wrong and why?"

Directions: The child's score is the sum of the highest number in each row. See previous description for what the letters are for.


0 A
1 B
2 C
3 D
4 E
What's wrong?
0 F
1 G
2 H
Why?
-1 J
0 K
1 L
2 M
3 N
3 O
4 P
4 Q
Justification of P(Orange wins a chip) > 1/2
1 R
2 S
Complete, systematic
1 T
2 U
Equiprobability of addends, factors, operators
1 V
Orange is more likely to win the whole game since he's more likely to win each chip

How would you fix the game?

	FROM	Score	
A'	A, B, C	0	Any change of the irrelevant physical features of the game. OR Any change of relevant features (e.g., the numbers on the spinners) for an irrelevant reason (e.g., it makes a more interesting game).
B'	G, H	+1	Any change (e.g., making the numbers on the spinners higher) that makes the game fairer (as opposed to making it more like more familiar spinner games).
C'		+1	Change of operation, replacing the × with another +, again designed to make the game fairer, (as opposed to making it more challenging computationally).
D'		+1	Changing the arrangement of the green and orange loops to make the distribution of obtainable numbers more evenly split between the two loops. This includes ideas like putting the even numbers in one loop and the odds in the other, even though this could not practically be shown on the existing board with only two loops of string.
E'		+1	Change of number of chips awarded (e.g., 2 chips to Green if the sum or product is in the green loop).
F'	B'	+2	Partial listing of how changes of the numbers on the spinners would create additional ways for Green to win. E.g., "If I change the 2s to 4s, then 4 & × & 4, and 4 & × & 5 will win for Green."; or some more general comment like "If the numbers on both spinners are 1 through 6, then multiplying will give bigger numbers that will win for Green" (see Subj. 5).
G'		+5	Complete enumeration of how proposed changes would affect the set of possible sums and products, even if equally likely occurrence of numbers in the various sections of the spinners is destroyed (e.g., changing just two of the three 2s on the spinner to 4s.)
H'	C'	+2	Partial listing of changes to operations, e.g., "60 / 3 will now win for Orange." (Note that this method of fixing is less likely in PSA C' than it was in PSA C, because subtraction and division would tend to result in lower

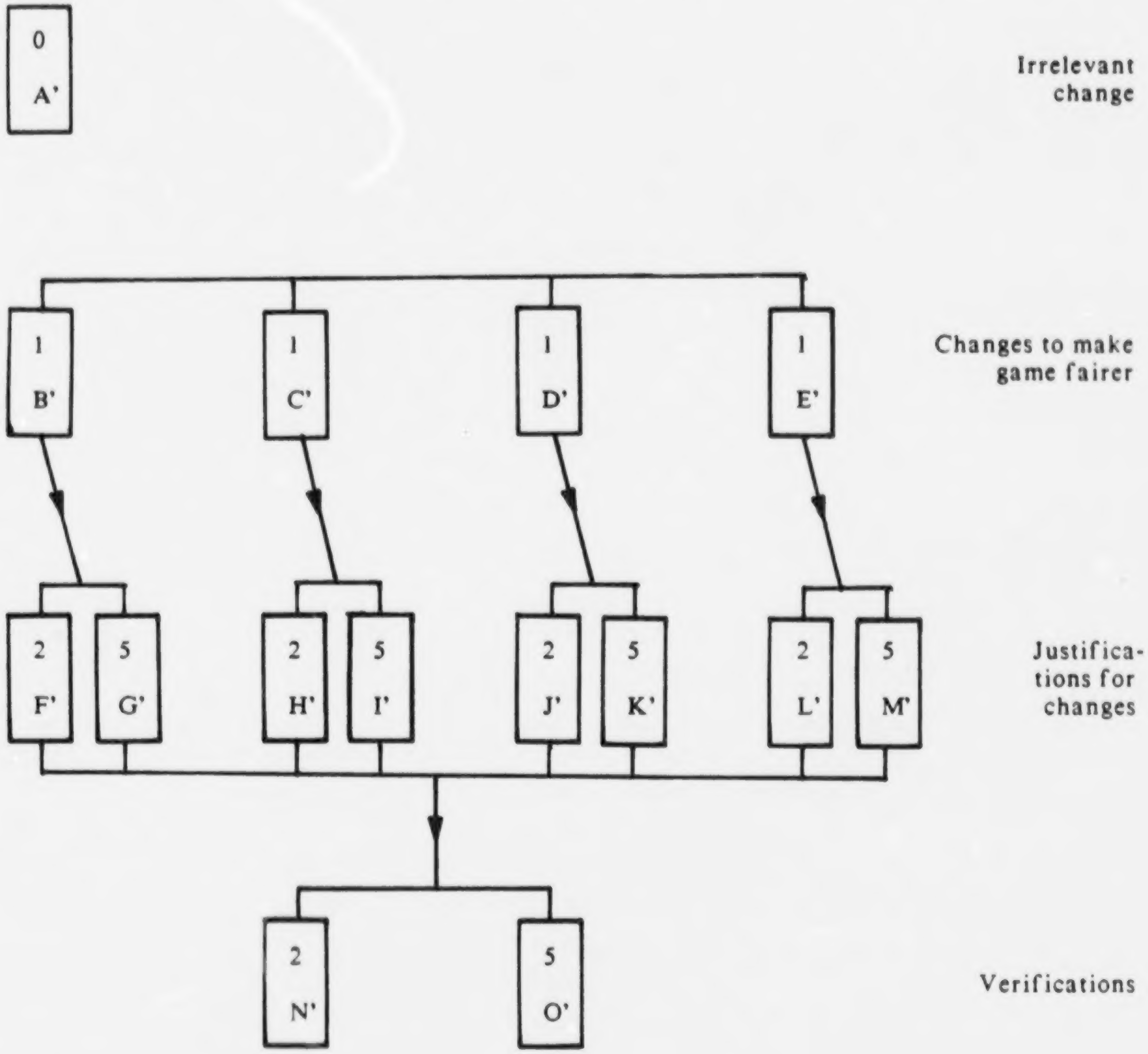
			numbers than addition and multiplication; and this would favor Orange even more. Hence the Subject might have to change the numbers on the spinners (e.g., 6 to 60) for this to be effective.)
I'		+5	Complete enumeration -- e.g., change + to ×; then Orange wins whenever 2 comes up and Green wins whenever 6 appears.
J'	D'	+2	Partial account -- e.g., "If the orange loops goes around the first three columns on the board, then 2 & + & 3; and 2 & × & 3 both win for Green."
K'		+5	Complete account -- e.g., "If the orange and green loops are moved like this, these are Green's winning spins: ..."
L'	E'	+2	Partial account -- e.g., "If it comes up 6 & × & 3, then Green will now get two chips instead of just one.
M'		+5	Complete account -- e.g., "Green will get two chips if 6 and × come up and...."

(Note: For some Ss, verification is part of justification.)

N'		+2	Verification that whatever suggestion has been made actually makes the game fairer, i.e. a verification that (at least in some cases) the game is now fairer than it was. (Note: It may be that the game is now biased toward the other player, but 2 points can still be awarded here.) (This includes an empirical justification -- playing the game again, with the changes, to see if it turns out to be fairer.)
O'		+5	Verification that the suggestion actually makes the game completely fair, i.e. a verification that Green and Orange are equally likely to win.

NOTE: Any repeated attempts at verification, followed by readjustment of numbers or whatever, would be reflected in the child's P-score, not here.

Flowchart for How would you fix the game?



APPENDIX II.I:

Statistics on Interrater Reliability

P-score reliability

The following table summarizes interrater reliability for the coding of the PSAs. The column on the left indicates the percent agreement between the two raters for each PSA. The column on the right indicates the results of Cohen's Kappa (Cohen, 1960), an adjusted measure of agreement between the raters.

	Mean Percent Agreement	Mean Kappa
PSA A*	93%	.55
PSA B*	94%	.55
PSA C*	89%	.44
Average	**92%**	**.51**

Percent agreement refers to the percentage of children for whom the two raters agreed. This figure was computed separately for each of the 17 actions and heuristics coded and then averaged across the 17 to arrive at a single figure for each PSA. As the table shows, raters agreed 92% of the time (on average) in coding the PSAs.

Readers may note the discrepancy between the percent agreement observed and the resulting Kappas obtained. This discrepancy is due to the nature of the data produced by the PSAs. Since each PSA lends itself to certain problem-solving actions and heuristics over others, each PSA resulted in a certain number of actions that were used by either all of the children in the sample or by none of them; for these actions, perfect agreement between raters would consist of all children being coded as using these particular actions or as not using them. However, the computation of Cohen's Kappa is such that a perfect score requires some variability in subjects' use of a behavior; that is, some subjects must be coded as exhibiting the behavior and others as not exhibiting it. When this variability is not present in subjects' behavior (as is often the case here), the Kappa formula results in a value near zero[1]; indeed, there were several cases in which perfect agreement between raters resulted in a Kappa of 0. Thus, the appropriateness of the Kappa statistic for data such as these (in which some behaviors were rated by both raters as being present in either all or none of the subjects) should be interpreted as very conservative estimates of agreement beyond chance, and it is included here as a standard measure of agreement.

M-score reliability

Because the scheme for creating M-scores is very different from that for P-scores (as is evident from a comparison of the coding guides presented in Appendices II.G and II.H), a different procedure was used for assessing reliability for M-scores. The following table presents the Pearson product-moment correlations between the scores given by the two raters.

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[1] In addition, there were three instances in which the lack of variability made the Kappa formula impossible to compute, as it would have required dividing by 0. These instances have been omitted from this analysis.

	Correlation
PSA A*	.92
PSA B*	.98
PSA C*	.97
Average	**.96**

As this table indicates, the scores given by the two raters were highly correlated ($p < .001$) in all cases.

APPENDIX II.J:

Viewers' and Nonviewers' Use of Goal II Behaviors

VIEWERS' AND NONVIEWERS USE OF GOAL II BEHAVIORS

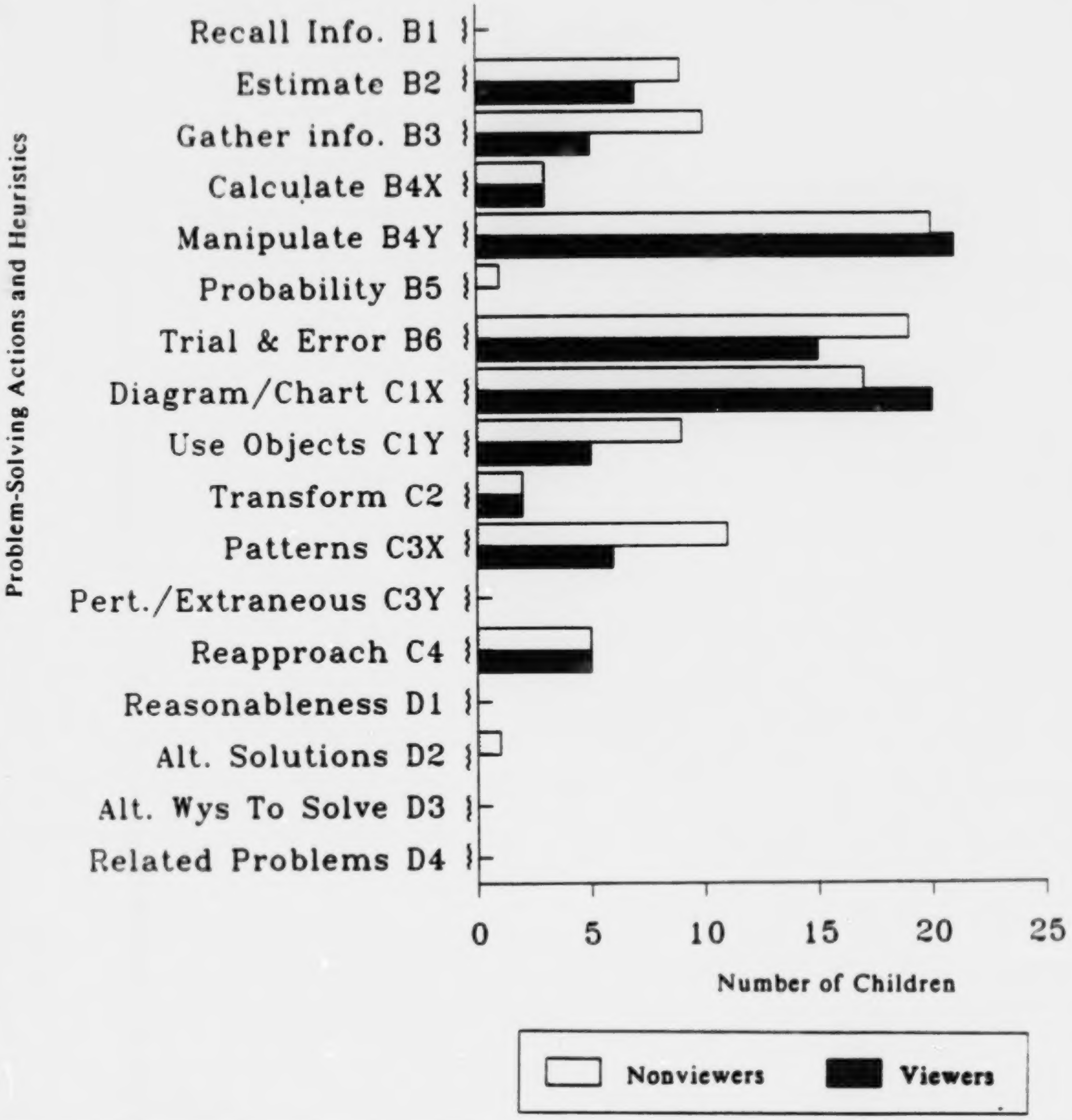
The following bar graphs represent the number of viewers and nonviewers who used each of the problem-solving actions and heuristics described under Goal II. There are two charts for each PSA, one representing performance in the pretest and the other representing the posttest.

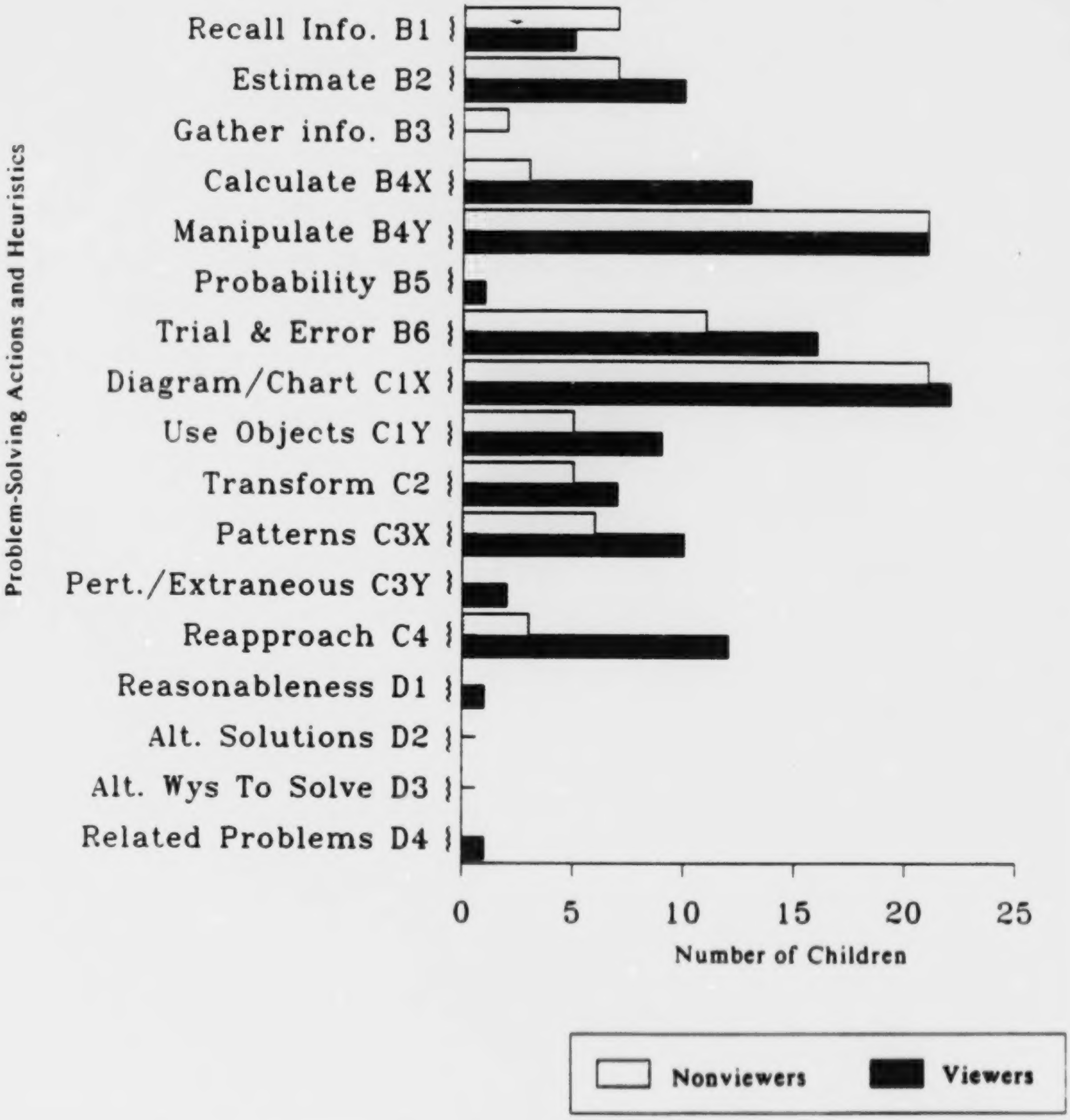
As these graphs demonstrate, both viewers' and nonviewers' distributions of actions and heuristics varied across the three PSAs. This is not surprising, considering that the three PSAs present problems that are very different from each other; while it might be extremely useful to consider probability, for example, in PSA C*, it would be less useful to do so in either of the other two PSAs.

More important, though, is a comparison of the performance of viewers and nonviewers within each PSA. For each PSA, the distribution of problem-solving actions and heuristics used by viewers and nonviewers is quite similar in the pretest (i.e. before the viewers watched **SQUARE ONE TV**). However, while nonviewers' distribution remained relatively unchanged in the posttest, viewers' distribution of behaviors had changed markedly. Different actions and heuristics had been enhanced in each PSA, as is discussed in Chapters 5 and 6.

PSA A* - PRETEST

Number of children using particular problem-solving actions and heuristics



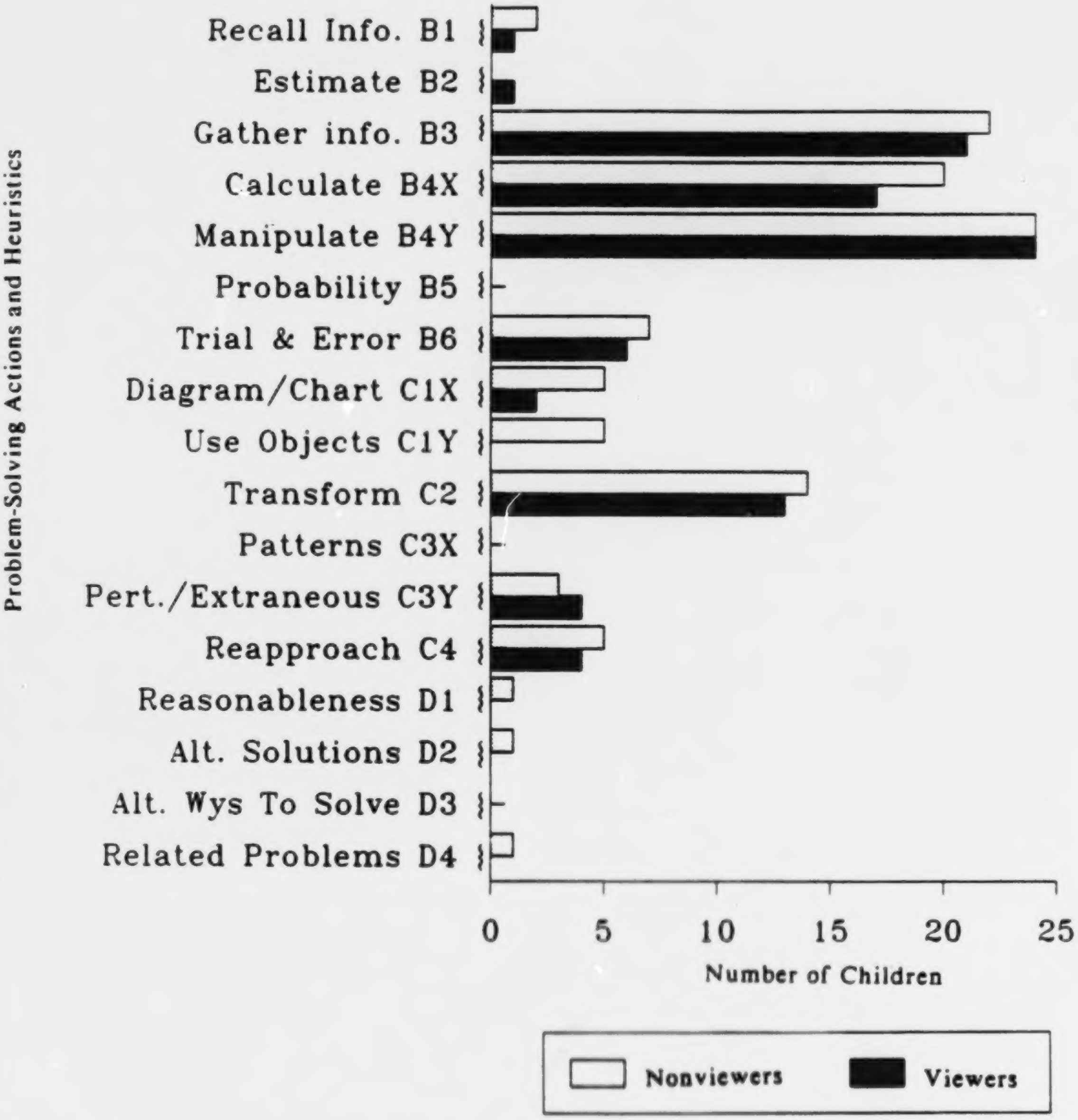


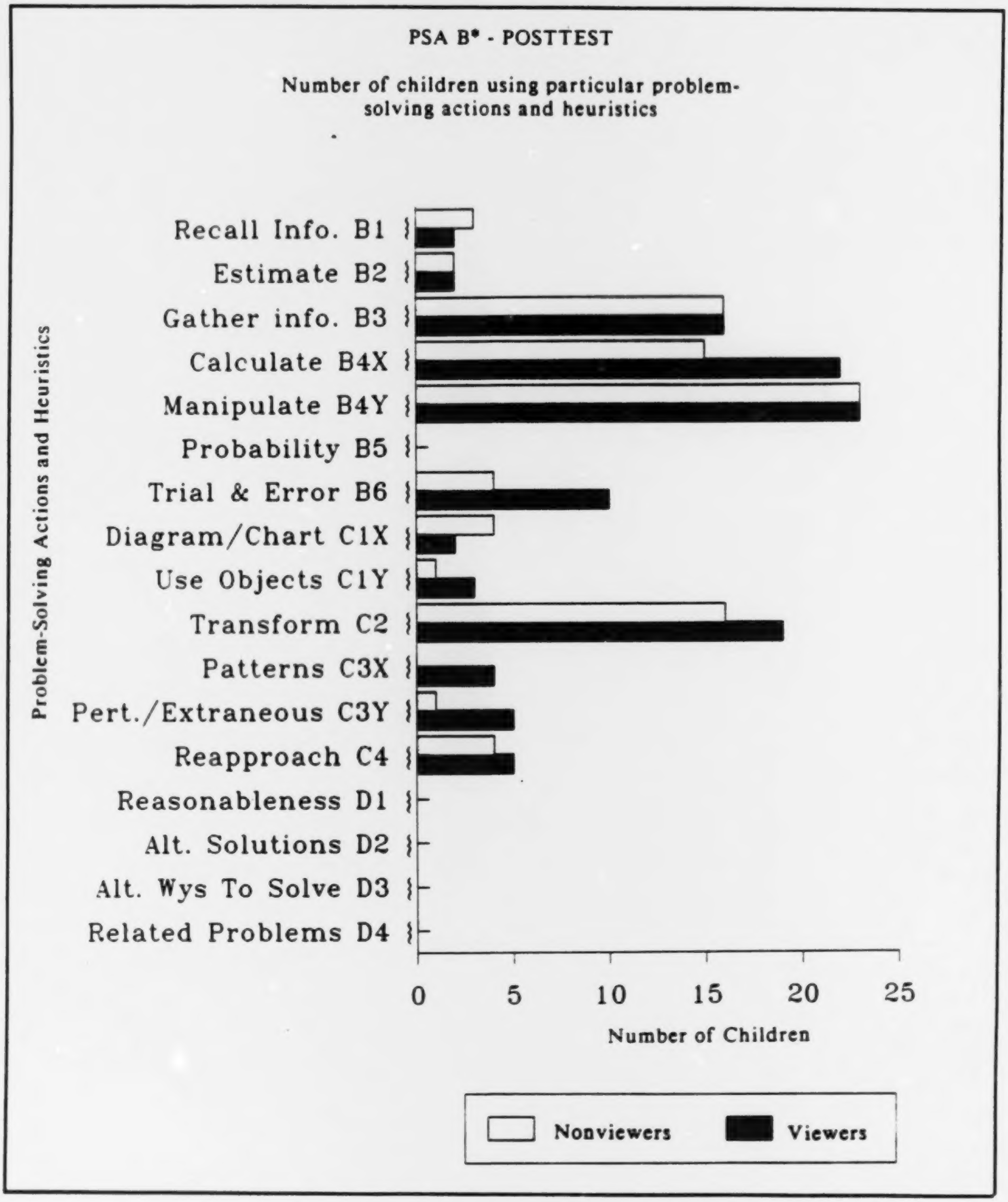
PSA A* - POSTTEST

Number of children using particular problem-solving actions and heuristics

PSA B* - PRETEST

Number of children using particular problem-solving actions and heuristics



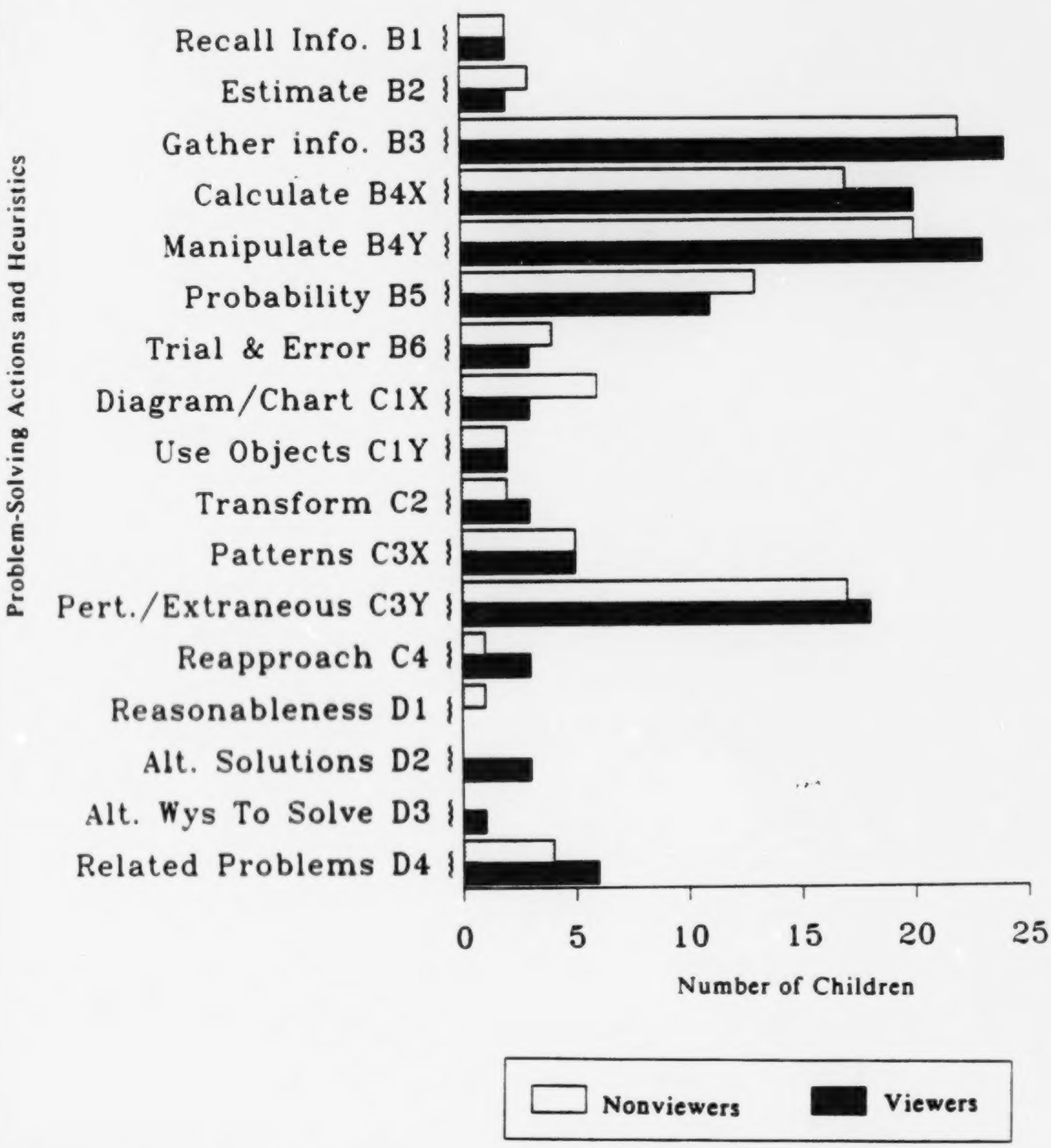


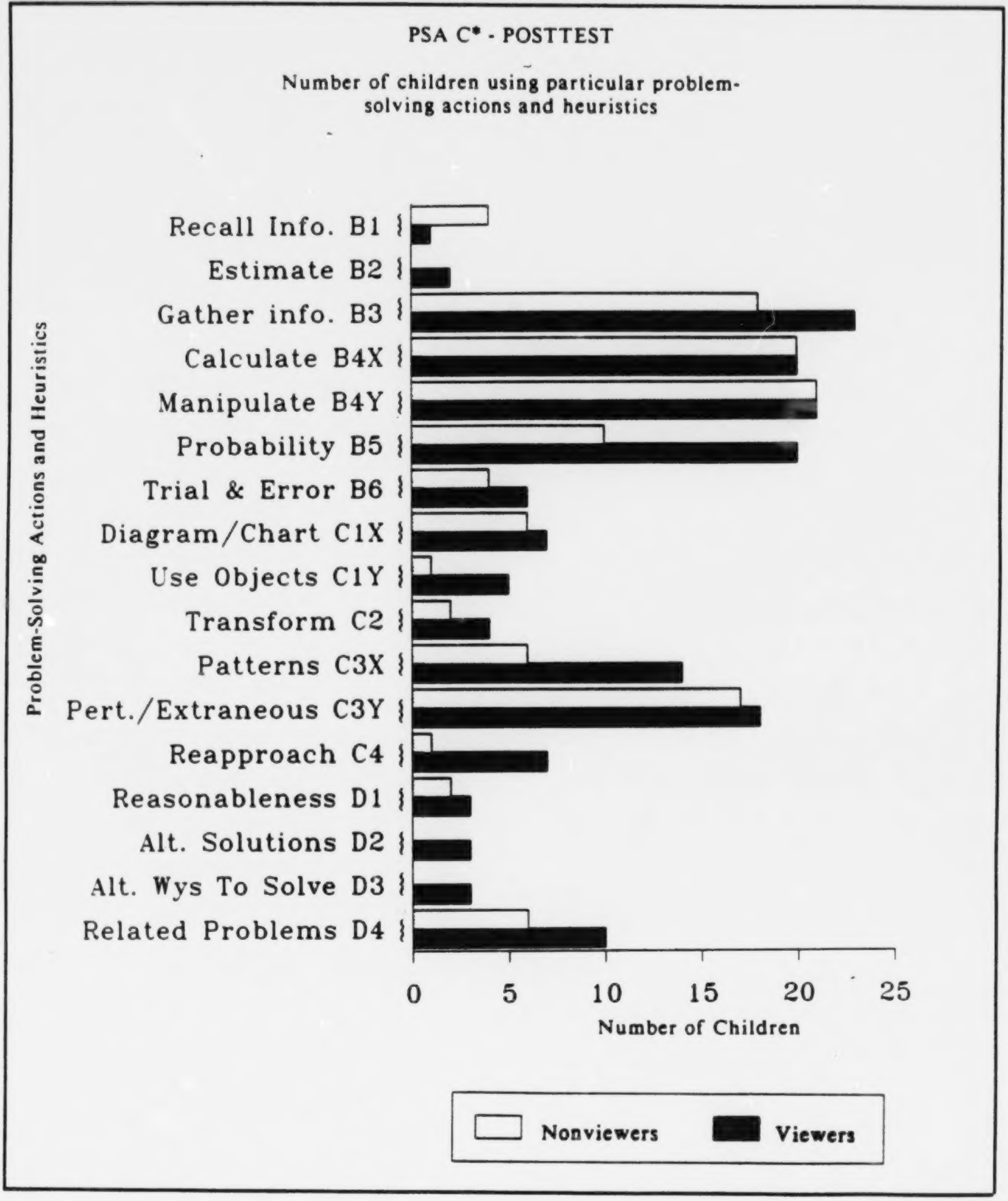
PSA B* - POSTTEST

Number of children using particular problem-solving actions and heuristics

PSA C* - PRETEST

Number of children using particular problem-solving actions and heuristics





PSA C* - POSTTEST

Number of children using particular problem-solving actions and heuristics

APPENDIX II.K:

Statistical Analyses Used in the Results Sections

STATISTICAL ANALYSES USED IN RESULTS SECTION

This Appendix presents information on the statistical analyses used in the Results section of this volume (Chapter 5). It is divided into sections similar to those used in the Results section; each section lists the relevant statistical analyses and their results.

For the convenience of the reader, each section is labeled with the page number to which it refers in the text.

p. 62 Comparison within pairs of PSAs

To determine whether prime and nonprime problems differed in difficulty, pretest data was analyzed via t-tests conducted within each pair of PSAs (e.g., between scores on PSAs A and A').

The mean P-scores obtained by all subjects are shown in the following table. Standard deviations are shown in parentheses.

	PSA A*	PSA B*	PSA C*
PRIME	8.44 (3.48)	9.80 (2.80)	12.88 (5.99)
NONPRIME	7.83 (2.59)	9.04 (2.87)	11.26 (5.20)

Pairwise t-tests revealed no significant differences within any of the pairs of PSAs (t_{46} = .70, N.S., for PSA A vs. A', t_{46} = .92, N.S., for PSA B vs. B', and t_{46} = 1.01, N.S., for PSA C vs. C').

The subjects' mean M-scores are shown in the following table. Standard deviations are shown in parentheses.

	PSA A*	PSA B*	PSA C*
PRIME	5.40 (1.41)	8.92 (1.87)	3.84 (4.40)
NONPRIME	4.96 (1.49)	9.26 (2.28)	5.91 (4.87)

Here, too, t-tests revealed no differences within any of the pairs of PSAs (t_{46} = 1.05, N.S., for PSA A vs. A', t_{46} = -.56, N.S., for PSA B vs. B', and t_{46} = -1.54, N.S., for PSA C vs. C').

Thus, because no differences were found for either P- or M-scores, the data were collapsed across primes and nonprimes for all subsequent analyses.

p. 62 Amount of time spent

For each PSA, a two-way analysis of variance was used to compare the amount of time that viewers and nonviewers spent working on the PSA; the factors used were Viewer/ Nonviewer (between subjects factor) and Pretest/Posttest (a within subjects factor).

The average number of seconds spent by the children are shown in the following tables. Standard deviations are shown in parentheses.

PSA A*

	Pretest	Posttest
VIEWERS	407 (175)	416 (169)
NONVIEWERS	488 (211)	424 (150)

PSA B*

	Pretest	Posttest
VIEWERS	365 (148)	362 (132)
NONVIEWERS	393 (170)	404 (184)

PSA C*

	Pretest	Posttest
VIEWERS	757 (385)	763 (385)
NONVIEWERS	656 (366)	778 (391)

All of the obtained F values were nonsignificant ($F_{1,44}$ = 1.13, N.S. for the interaction in PSA A*, $F_{1,44}$ =.04 for PSA B*, and $F_{1,44}$ = .61, N.S. for PSA C*).

p. 63 Comparison of P-scores, Row-score, and Column-scores

The three scores were compared through three pairwise correlations. The resulting correlation coefficients were: $r_{\text{P-score*Row-score}} = .98$, $r_{\text{P-score*Column-score}} = .91$, and $r_{\text{Row-score*Column-score}} = .91$. All three values are significant ($p < .001$). Because the three types of scores were almost perfectly correlated with each other, we decided to analyze only the children's P-scores, since these contained the greatest amount of information about their performance.

p. 64 Percentage of children showing P-score increases

For each PSA, a chi-square test of contingency was used to determine whether more viewers than nonviewers showed a gain in their P-scores from pretest to posttest. The differences between the two groups were significant for all three PSAs ($\chi^2 = 16.33$, $p < .001$ for PSA A*, $\chi^2 = 7.85$, $p < .01$ for PSA B*, and $\chi^2 = 5.37$, $p < .05$ for PSA C*).

p. 65 Analysis of P-scores: PSA A*

The mean P-scores obtained by viewers and nonviewers in PSA A* are shown in the following tables. Standard deviations are shown in parentheses.

PSA A*

	Pretest	Posttest
VIEWERS	7.33 (2.39)	11.00 (3.54)
NONVIEWERS	8.96 (3.48)	6.83 (2.14)

These data were analyzed via a two-way analysis of variance. The analysis revealed a significant interaction of Pretest/Posttest with Viewer/Nonviewer ($F_{1,44} = 30.16$, $p < .001$), indicating that viewers showed a greater pretest-posttest gain than nonviewers did. A Fisher Least Significant Difference indicated that both viewers and nonviewers changed significantly from pretest to posttest ($p < .001$ for viewers and $p < .01$ for nonviewers). Although nonviewers' scores were significantly higher than viewers' in the pretest ($p < .05$), viewers received significantly higher P-scores than nonviewers did in the posttest ($p < .01$).

In addition, a marginal main effect was found for Viewer/Nonviewer ($F_{1,44} = 3.59$, $p < .10$). However, because this effect collapses data across pretest and posttest, it is of little experimental interest and will not be discussed further. No main effect was observed for Pretest/Posttest ($F_{1,44} = 2.14$, N.S.).

p. 65 Analysis of P-scores: PSA B*

The mean P-scores obtained by viewers and nonviewers in PSA B* are shown in the following table. Standard deviations are shown in parentheses.

PSA B*

	Pretest	Posttest
VIEWERS	8.79 (2.52)	10.75 (3.49)
NONVIEWERS	10.08 (3.02)	7.92 (2.62)

A two-way analysis of variance revealed there to be a significant Pretest/Posttest x Viewer/Nonviewer interaction ($F_{1,44} = 18.63$, $p < .001$), indicating that viewers had shown a greater pretest-posttest gain than nonviewers did. A Fisher Least Significant Difference test indicated that both viewers and nonviewers changed significantly from pretest to posttest ($p < .01$). A significant difference was also found between viewers and nonviewers in the posttest ($p < .001$). The pretest difference between the two groups was only marginal ($p < .10$).

No significant main effect was found for either Pretest/Posttest ($F_{1,44} = .05$, N.S.) or Viewer/Nonviewer ($F_{1,44} = 1.21$, N.S.).

p. 66 Analysis of P-scores: PSA C*

The mean P-scores obtained by viewers and nonviewers are shown in the following table. Standard deviations are shown in parentheses.

	Pretest	Posttest
VIEWERS	11.67 (5.31)	17.75 (7.53)
NONVIEWERS	12.54 (5.90)	11.25 (4.99)

A two-way analysis of variance revealed there to be a significant Pretest/Posttest x Viewer/Nonviewer interaction ($F_{1,44} = 18.46$, $p < .001$), indicating that viewers had shown a greater pretest-posttest gain than nonviewers did. A Fisher Least Significant Difference test indicated that viewers changed significantly from pretest to posttest ($p < .001$) but nonviewers did not. No significant difference was found between the two groups in the pretest, but a significant difference was found between viewers and nonviewers in the posttest ($p < .001$).

A significant main effect was found for Pretest/ Posttest ($F_{1,44} = 7.79$, $p < .01$), and a marginal one found for Viewer/Nonviewer ($F_{1,44} = 3.48$, $p < .10$). However, because the Viewer/Nonviewer difference collapses data across pretest and posttest and the Pretest/ Posttest difference collapses data across experimental groups, they are of little experimental interest and will not be discussed further.

p. 67 Relation between P-scores and standardized mathematics scores

As noted in the text, two sets of analyses were conducted here: one assessing the degree to which children's performance on a standardized mathematics test was related to their P-scores (in the absence of any treatment), and one examining the degree to which standardized mathematics test scores were related to viewers' change in performance from pretest to posttest.

To examine the first issue, correlations were run between children's scores on a standardized mathematics test administered by the school and the P-scores they obtained in the pretest; posttest scores were not included since viewing **SQUARE ONE TV** might increase PSA performance, thus changing the relationship between PSA performance and standardized mathematics scores.

The children's P-scores were not significantly related to their scores on a standardized mathematics test ($r = .09$, N.S., for PSA A*, $r = -.18$, N.S., for PSA B*, and $r = .11$, N.S., for PSA C*).

To examine the second issue (the relationship between performance on standardized mathematics tests and viewers' increase from pretest to posttest), correlations were run between viewers' standardized mathematics test scores and their pretest-posttest change scores (i.e. each child's posttest score minus his or her pretest score).

These correlations revealed that viewer's change scores were not significantly related to their scores on standardized mathematics tests ($r = .00$, N.S., for PSA A*, $r = .26$, N.S. for PSA B*, and $r = -.26$, N.S., for PSA C*).

Additionally, the reader may be interested to note that even nonviewers' change scores were not significantly related to their standardized mathematics scores either ($r = .01$, N.S., for PSA A*, $r = .24$, N.S., for PSA B*, and $r = -.09$, N.S.).

p. 68 Combined P-scores

A necessary prerequisite for combining P-scores from the three PSAs was that they be significantly correlated with each other. Correlations performed between the children's P-scores on PSAs A*, B*, and C* showed that this prerequisite had been satisfied. The correlation coefficients were: $r_{A^*B^*} = .37$ ($p < .01$), $r_{A^*C^*} = .52$ ($p < .001$), and $r_{B^*C^*} = .45$ ($p < .01$).

P-scores from PSAs A*, B*, and C* were combined into a weighted sum, using a principal components analysis to derive appropriate weights. The equation used to produce the combined score was:

$$\text{Combined P-score} = .304P_{A^*} + .207P_{B^*} + .930P_{C^*}$$

The mean combined P-scores obtained by viewers and nonviewers are shown in the following table. Standard deviations are shown in parentheses.

COMBINED P-SCORES

	Pretest	Posttest
VIEWERS	14.90 (5.47)	22.08 (7.40)
NONVIEWERS	16.47 (6.55)	14.18 (5.14)

A two-way analysis of variance revealed there to be a significant Pretest/Posttest × Viewer/Nonviewer interaction ($F_{1,44} = 35.74$, $p < .001$), indicating that viewers had shown a greater pretest-posttest gain than nonviewers did. A Fisher Least Significant Difference test indicated that viewers had changed significantly from pretest to posttest ($p < .001$) and that their posttest scores were significantly higher than nonviewers' ($p < .001$). The pretest difference between the two groups was found to be marginal ($p < .10$), as was the control group's change from pretest to posttest.

Again, a significant main effect was found for Pretest/ Posttest ($F_{1,44} = 6.32$, $p < .05$). However, because this difference collapses data across experimental groups, it is of little experimental interest and will not be discussed further. A significant main effect was not observed for Viewer/Nonviewer ($F_{1,44} = 3.38$, N.S.).

p. 69 Effects of sex, ethnicity, and SES

A three-way analysis of variance was performed on children's combined P-scores, using Pretest/Posttest, Viewer/Nonviewer, and Sex as factors. The children's mean P-scores are shown in the following tables. Standard deviations are shown in parentheses.

COMBINED P-SCORES

BOYS

	Pretest	Posttest
VIEWERS	15.53 (6.65)	21.43 (9.03)
NONVIEWERS	14.44 (5.40)	13.99 (3.57)

GIRLS

	Pretest	Posttest
VIEWERS	14.27 (4.17)	22.73 (5.66)
NONVIEWERS	18.51 (7.17)	14.37 (6.51)

The analysis revealed no main effect of Sex ($F_{1,44} = .48$, N.S.), but a marginal three-way interaction was found between Pretest/Posttest, Viewer/Nonviewer, and Sex ($F_{1,44} = 3.81$, $p < .10$). Post hoc analyses revealed that among viewers, both boys and girls showed a significant increase from pretest to posttest ($p < .01$ for boys and $p < .001$ for girls), but that while male nonviewers' scores did not change significantly from pretest to posttest, female nonviewers' scores showed a significant drop ($p < .05$). Thus, the data indicate that the three-way interaction does not reflect a differential effect of exposure to **SQUARE ONE TV** among viewers but rather points to a differential decline among nonviewers.

A second three-way analysis of variance was performed, using Pretest/Posttest, Viewer/Nonviewer, and Ethnicity as factors. The children's mean combined P-scores are shown in the following tables. Standard deviations are shown in parentheses.

MINORITY

	Pretest	Posttest
VIEWERS	13.81 (4.21)	18.99 (5.81)
NONVIEWERS	17.12 (5.69)	13.17 (5.25)

NONMINORITY

	Pretest	Posttest
VIEWERS	17.55 (7.46)	29.57 (5.26)
NONVIEWERS	14.91 (8.60)	16.64 (4.21)

Ethnicity was found to have a significant main effect ($F_{1,44} = 5.78$, $p < .05$) in that nonminority children obtained higher P-scores than minority children overall. However, the three-way Pretest/Posttest× Viewer/ Nonviewer× Ethnicity interaction was not significant ($F_{1,44} = .13$, N.S.), indicating that the effect of **SQUARE ONE TV** did not differ for minority and nonminority children.

A third three-way analysis of variance was performed, using Viewer/Nonviewer, Pretest/Posttest, and SES as factors. The children's mean combined P-scores are shown in the following tables. Standard deviations are shown in parentheses.

COMBINED P SCORE

LOW SES

	Pretest	Posttest
VIEWERS	12.13 (3.30)	18.22 (6.24)
NONVIEWERS	16.68 (6.15)	12.23 (4.94)

MIDDLE SES

	Pretest	Posttest
VIEWERS	17.67 (5.90)	25.93 (6.57)
NONVIEWERS	16.27 (7.19)	16.13 (4.75)

SES was found to have a significant main effect ($F_{1,44} = 8.27$, $p < .01$) in that middle-SES children obtained higher P-scores than lower-SES children overall. However, the three-way Pretest/Posttest× Viewer/Nonviewer× SES interaction was not significant ($F_{1,44} = 1.35$, N.S.), indicating that the effect of **SQUARE ONE TV** did not differ for middle- and lower-SES children.

p. 70 Percentage of children showing M-score increases

As with P-scores, for each PSA, a chi-square test of contingency was used to determine whether more viewers than nonviewers showed a gain in their M-scores from pretest to posttest. The difference between the two groups was significant for both PSA A* ($\chi^2 = 8.39$, $p < .01$) and PSA C* ($\chi^2 = 30.14$, $p < .001$). However, this difference was not statistically greater than chance ($\chi^2 = 2.02$, N.S.); this lack of significance is apparently due to ceiling effects in the data, as will be discussed further when we turn to a more detailed discussion of children's M-scores in PSA B*.

p. 71 Number of orders produced in PSA A*

As noted in the text, no significant effects were found with respect to the number of orders produced in PSA A*. The the mean numbers of orders produced by viewers and nonviewers are shown in the following table. Standard deviations are shown in parentheses.

PSA A*: NUMBER OF ORDERS

	Pretest	Posttest
VIEWERS	11.00 (6.85)	14.30 (8.80)
NONVIEWERS	13.00 (6.29)	14.04 (8.14)

A two-way analysis of variance revealed no significant main effect of either Pretest/Posttest ($F_{1,92}$ = 1.67, N.S.) or Viewer/Nonviewer ($F_{1,92}$ = 1.05, N.S.) and no significant interaction between the two ($F_{1,92}$ = .42, N.S.).

p. 71 Analysis of M-scores: PSA A*

The mean M-scores obtained by viewers and nonviewers in PSA A* are shown in the following table. Standard deviations are shown in parentheses.

PSA A*

	Pretest	Posttest
VIEWERS	4.75 (1.07)	6.21 (1.32)
NONVIEWERS	5.63 (1.66)	5.58 (1.84)

A two-way analysis of variance revealed there to be a significant Pretest/Posttest× Viewer/Nonviewer interaction ($F_{1,44}$ = 9.07, $p < .005$), indicating that viewers had shown a greater pretest-posttest gain than nonviewers did. A Fisher Least Significant Difference test indicated that viewers changed significantly from pretest to posttest ($p < .001$) but nonviewers did not. Nonviewers received significantly higher M-scores than viewers in the pretest ($p < .05$). However, viewers received higher M-scores than nonviewers in the posttest; the difference between the two groups in the posttest was marginal ($p < .10$).

A significant main effect was found for Pretest/ Posttest ($F_{1,44} = 8.09$, $p < .01$). However, because this effect collapses data across experimental groups, it is of little experimental interest and will not be discussed further. No main effect was observed for Viewer/Nonviewer ($F_{1,44} = .12$, N.S.).

p. 73 Analysis of M-scores: PSA B*

The mean M-scores obtained by viewers and nonviewers in PSA B* are shown in the following table. Standard deviations are shown in parentheses.

PSA B*

	Pretest	Posttest
VIEWERS	9.29 (2.16)	9.29 (1.99)
NONVIEWERS	8.88 (1.99)	8.46 (2.19)

A two-way analysis of variance revealed no significant main effect of either Pretest/Posttest ($F_{1,44} = 1.46$, N.S.) or Viewer/Nonviewer ($F_{1,44} = .47$, N.S.) and no significant interaction between the two ($F_{1,44} = .47$, N.S.).

p. 74 Analyses of M-scores: PSA C*

The mean M-scores obtained by viewers and nonviewers in PSA C* are shown in the following table. Standard deviations are shown in parentheses.

PSA C*

	Pretest	Posttest
VIEWERS	4.29 (4.37)	10.21 (6.98)
NONVIEWERS	5.37 (5.05)	4.62 (5.04)

A two-way analysis of variance revealed there to be a significant Pretest/Posttest × Viewer/Nonviewer interaction ($F_{1,44} = 19.13$, $p < .001$), indicating that viewers had shown a greater pretest-posttest gain than nonviewers did. A Fisher Least Significant Difference test indicated that viewers changed significantly from pretest to posttest ($p < .001$) but nonviewers did not. A significant difference was also found between viewers and nonviewers in the posttest ($p < .001$).

A significant main effect was found for Pretest/ Posttest ($F_{1,44} = 11.49$, $p < .005$). However, because this effect collapses data across experimental groups, it is of little experimental interest and will not be discussed further. No main effect was observed for Viewer/Nonviewer ($F_{1,44} = 2.68$, N.S.).

p. 75 Relation between M-scores and standardized mathematics scores

As discussed above with regard P-scores, two issues were of interest here: the degree to which performance on standardized mathematics tests was related to performance in the pretest (i.e. before any treatment was administered) and the degree to which it was related to pretest-posttest change among viewers (i.e. the degree to which it interacted with the effects of viewing **SQUARE ONE TV**).

To examine the first issue, pairwise correlations were computed between children's performance on standardized mathematics tests and the M-scores they obtained in the pretest. These correlations revealed that the children's M-scores were not significantly related to either their standardized mathematics test scores ($r = -.06$, N.S., for PSA A*, $r = -.07$, N.S., for PSA B*, and $r = .02$, N.S., for PSA C*).

To examine the second issue (the relationship between performance on standardized mathematics tests and viewers' increase from pretest to posttest), correlations were run between viewers' standardized mathematics test scores and their pretest-posttest change scores (i.e. each child's posttest score minus his or her pretest score). These correlations revealed that viewer's change scores were not significantly related to their scores on standardized mathematics tests ($r = -.11$, N.S., for PSA A*, $r = -.11$, N.S., for PSA B*, and $r = .06$, N.S., for PSA C*).

Additionally, the reader may be interested to note that even nonviewers' change scores on PSA B* and PSA C* were not significantly related to their standardized mathematics scores ($r = .02$, N.S., for PSA B*, and $r = -.05$, N.S.) and only a marginal correlation was found between nonviewers' standardized mathematics test scores and their change scores on PSA A* ($r = .36$, $p < .10$).

p. 75 Combined M-scores

The combined M-score consists simply of the sum of the M-scores a child obtained in PSAs A*, B*, and C*. (However, see the next section for caveats concerning these combined scores.) The viewers' and nonviewers' mean combined M-scores are shown in the following table. Standard deviations are shown in parentheses.

COMBINED M-SCORES

	Pretest	Posttest
VIEWERS	18.33 (5.47)	25.71 (6.79)
NONVIEWERS	19.88 (6.74)	18.67 (6.79)

As noted in the text, the combined M-scores are presented as a summary statistic only (for reasons described below). Thus, no inferential statistical analyses were performed on these scores.

p. 77 Comparison of M-scores in PSAs A^*, B^*, and C^*

As we did for P-scores, pairwise correlations were performed among children's M-scores in the three PSAs. Unlike P-scores, however, not all of the three M-scores were significantly related to each other. M-scores in PSAs A^* and B^* were significantly related to each other ($r = .40$, $p < .02$), but there was only a marginal correlation between M-scores in PSAs A^* and C^* ($r = .25$, $p < .10$), and the correlation between M-scores in PSAs B^* and C^* was not significant ($r = .18$, N.S.). For this reason (as well as the conceptual reason discussed in the text), these scores are presented only a summary for the reader; we did not use combined M-scores in the sex, ethnicity, and SES analyses (presented below), although we had used combined scores in our P-score analyses.

p. 77 Effects of sex, ethnicity, and SES

A three-way Pretest/Posttest × Viewer/Nonviewer × Sex analysis of variance was conducted for each of the three PSAs. The children's mean M-scores are shown in the following tables. Standard deviations are shown in parentheses.

M-SCORES

PSA A^*: BOYS

	Pretest	Posttest
VIEWERS	4.75 (1.36)	6.42 (1.08)
NONVIEWERS	5.50 (1.09)	5.42 (1.62)

PSA A^*: GIRLS

	Pretest	Posttest
VIEWERS	4.75 (0.75)	6.00 (1.54)
NONVIEWERS	5.75 (2.14)	5.75 (2.09)

PSA B*: BOYS

	Pretest	Posttest
VIEWERS	9.08 (2.28)	9.08 (2.11)
NONVIEWERS	8.42 (2.02)	7.75 (1.82)

PSA B*: GIRLS

	Pretest	Posttest
VIEWERS	9.50 (2.11)	9.50 (1.93)
NONVIEWERS	9.33 (1.92)	9.17 (2.37)

PSA C*: BOYS

	Pretest	Posttest
VIEWERS	4.08 (4.46)	11.75 (8.06)
NONVIEWERS	4.25 (4.85)	4.83 (5.67)

PSA C*: GIRLS

	Pretest	Posttest
VIEWERS	4.50 (4.46)	8.67 (5.63)
NONVIEWERS	6.50 (5.20)	4.42 (4.56)

No significant effect of Sex was found in any of the three PSAs, either as a main effect ($F_{1,88} = .02$, N.S., for PSA A*, $F_{1,88} = .23$, N.S., for PSA B*, and $F_{1,88} = .57$, N.S., for PSA C*) or in the three-way interactions ($F_{1,88} = .16$, N.S., for PSA A*, $F_{1,88} = .47$, N.S., for PSA B*, and $F_{1,88} = 2.42$, N.S., for PSA C*).

A second three-way analysis of variance was performed, using Pretest/Posttest, Viewer/Nonviewer, and Ethnicity as factors. The children's mean combined P-scores are shown in the following tables. Standard deviations are shown in parentheses.

M-SCORES

PSA A*: MINORITIES

	Pretest	Posttest
VIEWERS	4.82 (1.19)	6.18 (1.13)
NONVIEWERS	5.47 (1.51)	5.12 (1.80)

PSA A*: NONMINORITIES

	Pretest	Posttest
VIEWERS	4.57 (1.80)	6.29 (1.60)
NONVIEWERS	6.00 (1.50)	6.71 (1.78)

PSA B*: MINORITIES

	Pretest	Posttest
VIEWERS	9.47 (2.18)	9.29 (1.93)
NONVIEWERS	8.94 (1.92)	8.18 (1.98)

PSA B*: NONMINORITIES

	Pretest	Posttest
VIEWERS	8.86 (2.19)	9.29 (2.29)
NONVIEWERS	8.71 (2.29)	9.14 (2.67)

PSA C*: MINORITIES

	Pretest	Posttest
VIEWERS	3.29 (3.58)	7.53 (5.26)
NONVIEWERS	6.24 (5.14)	4.65 (4.50)

PSA C*: NONMINORITIES

	Pretest	Posttest
VIEWERS	6.71 (5.41)	16.71 (6.58)
NONVIEWERS	3.29 (4.46)	4.57 (6.58)

No significant main effect was found for Ethnicity in either PSA A* ($F_{1,44}$ = 1.67, N.S.) or PSA B* ($F_{1,44}$ = 0.00, N.S.), and a marginally significant main effect was found in PSA C* ($F_{1,44}$ = 3.00, $p < .10$). In no case, however, was the three-way Pretest/Posttest × Viewer/ Nonviewer × Ethnicity interaction found to be significant ($F_{1,44}$ = .42, N.S., for PSA A*, $F_{1,44}$ = .19, N.S., for PSA B*, and $F_{1,44}$ = .85, N.S., for PSA C*); thus, the data indicate that the effects of **SQUARE ONE TV** did not differ for minority and nonminority children.

Finally, a three-way Pretest/Posttest × Viewer/ Nonviewer × SES analysis of variance was conducted for each of the three PSAs. The results are shown in the following tables. Standard deviations are shown in parentheses.

M-SCORE

PSA A*: LOW SES

	Pretest	Posttest
VIEWERS	4.50 (1.17)	5.75 (0.87)
NONVIEWERS	5.08 (1.38)	4.67 (1.16)

PSA A*: MIDDLE SES

	Pretest	Posttest
VIEWERS	5.00 (0.95)	6.67 (1.56)
NONVIEWERS	6.17 (1.80)	6.50 (1.98)

PSA B*: LOW SES

	Pretest	Posttest
VIEWERS	9.17 (2.41)	9.58 (1.98)
NONVIEWERS	8.67 (1.67)	8.00 (1.86)

PSA B*: MIDDLE SES

	Pretest	Posttest
VIEWERS	9.42 (1.98)	9.00 (2.05)
NONVIEWERS	9.08 (2.31)	8.92 (2.47)

PSA C*: LOW SES

	Pretest	Posttest
VIEWERS	2.17	6.17
	(2.69)	(5.18)
NONVIEWERS	5.42	4.50
	(4.70)	(4.80)

PSA C*: MIDDLE SES

	Pretest	Posttest
VIEWERS	6.42	14.25
	(4.78)	(6.27)
NONVIEWERS	5.33	4.75
	(5.58)	(5.48)

No significant effect of SES was found for PSA B*, either as a main effect ($F_{1,44} = .23$, N.S.) or in the three-way interaction ($F_{1,44} = 1.17$, N.S.). SES was found to exert a significant main effect in PSA A* ($F_{1,44} = 11.48$, $p < .01$) and PSA C* ($F_{1,44} = 6.32$, $p < .05$), but SES did not interact with the effect of watching **SQUARE ONE TV** in either of these two PSAs ($F_{1,44} = .11$, N.S., and $F_{1,88} = 1.35$, N.S., respectively); thus, in both cases, the results suggested that the effects of **SQUARE ONE TV** did not differ for middle- and lower-SES children.

p. 77 Relation between P-scores and M-scores

The correlation between P- and M-scores was found to be .52 ($p < .001$).

p. 78 New behaviors

For each PSA, t-tests were performed to compare the proportions of new behaviors produced by viewers and nonviewers. The scores used in the analysis were arcsine transforms of the proportion:

$$\frac{\text{Number of new posttest behaviors}}{\text{Total number of pretest + posttest behaviors}}$$

The mean proportions of new behaviors produced by viewers and nonviewers are shown in the following table. Standard deviations are shown in parentheses.

NEW POSTTEST BEHAVIORS

	PSA A*	PSA B*	PSA C*
VIEWERS	.427 (.257)	.325 (.230)	.494 (.207)
NONVIEWERS	.215 (.165)	.231 (.144)	.312 (.164)

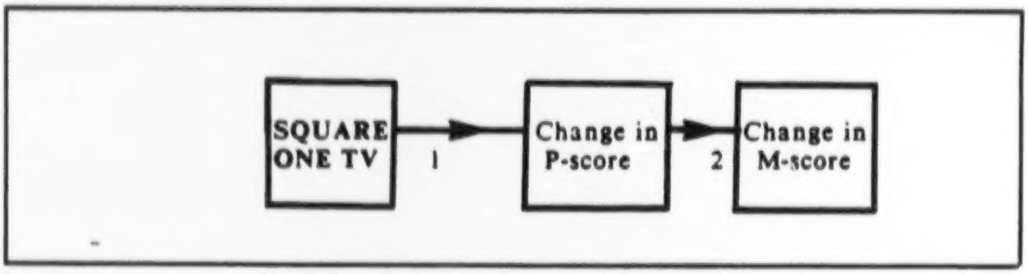
Significant differences between viewers and nonviewers were found for PSA A* ($t_{39} = 3.39, p < .01$) and PSA C* ($t_{43} = 3.37, p < .01$). The difference between the two groups was marginal in PSA B* ($t_{38} = 1.69, p < .10$).

p. 86 Relationship among SQUARE ONE TV, P-scores, and M-scores

In an attempt to establish a model of the effects of **SQUARE ONE TV** on children's P-scores and M-scores, a model-fitting analysis was conducted using the structural equations computer program EQS (Bentler, 1985). For this analysis, change scores were computed by subtracting each child's pretest P-score and M-score on each PSA from its posttest counterpart. All change scores were then standardized to compensate for the fact that P-scores and M-scores employed different scales (recall that M-scores had upper limits in that, for each PSA, there was a maximum possible M-score that could not be exceeded; P-scores had no such limits).

A 3 × 3 covariance matrix was formed for each PSA. The variables in each matrix were Condition (Viewer/Nonviewer), standardized Change in P-score, and standardized Change in M-score. Two path models were fitted to each covariance matrix; we present the path diagrams and results of the model-fitting procedures in the figures below. Each figure contains two goodness-of-fit indices: the χ^2 statistic and the Bentler-Bonett normed fit index (NFI). Parameter estimates, which may be interpreted here as partial regression coefficients, are presented in their unstandardized (a) and standardized (A) forms.

As described in Chapter 6, the first model tested was the simple causal model:



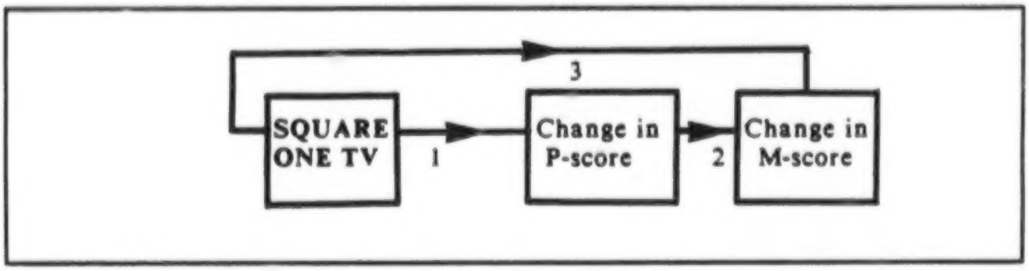
The results of the model-fitting procedure were as follows:

PSA	χ^2(1 df)	NFI	a_1	a_2	A_1	A_2
A*	1.74	.95	1.25*	0.43*	.63	.43
B*	0.02	.99	1.06*	0.15	.54	.15
C*	5.78*	.86	1.06*	0.58*	.54	.58

Note: Values marked with an asterisk (*) are statistically significant ($p < .05$).

This model fit the data well for PSA A* and PSA B*, as indicated by NFI values higher than .90 and the nonsignificant χ^2 obtained in each case. It did not fit the data for PSA C*, however, as reflected in the lower NFI value and a significant χ^2; an examination of residuals from the model-fitting procedure revealed a marked discrepancy between the observed covariance between Condition and Change in M-score and the estimate produced by EQS.

We next tested a somewhat more refined model:



For this model to have more data than free parameters to be estimated, the variances of the errors in predicting the two change scores were constrained to be equal. The results of the second model-fitting procedure were as follows:

PSA	χ^2(1 df)	NFI	a_1	a_2	a_3	A_1	A_2	A_3
A	0.796	.977	1.246*	0.294*	0.437	.603	.320	.230
B	1.162	.932	1.063*	0.140	0.051	.504	.160	.028
C	0.413	.990	1.059*	0.399*	0.652*	.552	.377	.321

Note: Values marked with an asterisk (*) are statistically significant ($p < .05$).

This second model fit the data well for all three PSAs, as reflected in NFI values above .90 and the nonsignificant χ^2 values obtained in each case.

p. 90 Motivation

The correlation between the children's Magnitude scores in Motivation and their combined P-scores in the posttest was found to be .42 ($p < .01$).